Equivalent Matrices & Multiplication

Matrices are equivalent if

a) same size

b) corresponding elements are =

$$\begin{bmatrix} 3 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 12 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 5 & 12 \\ -4 & 7 \end{bmatrix}$$

Solve for EACH variable.

$$\begin{bmatrix} 4x-1 & 14 \\ -12 & 4y \end{bmatrix} = \begin{bmatrix} 3x+10 & 14 \\ w-12 & 20 \end{bmatrix}$$

$4y = 20$

$\boxed{y=5}$

$4x-1 = 3x+10$

$\boxed{x=11}$

$-12 = w-12$

$\boxed{0=w}$

$$\begin{bmatrix} -3 & x+4 & 8 \\ x+y & 2 & 0 \\ z^2 & 1/3v & 1 \end{bmatrix} = \begin{bmatrix} -3 & 16 & 8 \\ 10 & 2 & 4w \\ 25 & 3 & 1 \end{bmatrix}$$

$z^2 = 25$

$\boxed{z = \pm 5}$

$x+4=16$

$\boxed{x=12}$

$x+y=10$

$12+y=10$

$\boxed{y=-2}$

$0=4w$

$\boxed{0=w}$

$\frac{1}{3}v = 3$

$\boxed{v=9}$

Multiplying Matrices

$$\begin{bmatrix} -1 & 3 \\ 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 23 \\ 28 & 2 \end{bmatrix}$$

M

$2\times2 \quad 2\times2$ New

yes

$m_{11}: -1(4) + 3(2) = 2$

$m_{12}: (-1)(-2) + 3(7) = 23$

$m_{21}: 6(4) + 2(2) = 28$

$m_{22}: 6(-2) + 2(7) = 2$

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ -3 & 1 \\ p & q \end{bmatrix} = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$$

M

New $2\times3 \quad 3\times2$

yes

$m_{11}: a(2) + -3b + cp$

$m_{12}: 4a + b + cq$

$m_{21}: 2d - 3e + fp$

$m_{22}: 4d + e + fq$

Using Matrices to Solve Systems of Equations

$x - 2 = 5$

$+2 \quad +2$

$x = 7$

$$X - \begin{bmatrix} 3 & -5 \\ -9 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ -1 & 7 \end{bmatrix}$$

$$+\begin{bmatrix} 3 & -5 \\ -9 & 0 \end{bmatrix} + \begin{bmatrix} 3 & -5 \\ -9 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 7 & 1 \\ -10 & 7 \end{bmatrix}$$

$x + 3 = 9$

$-3 \quad -3$

$x = 6$

$$X + \begin{bmatrix} -2 & 3 \\ 1 & 0 \\ -4 & -8 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ -5 & -3 \\ 6 & -8 \end{bmatrix}$$

$$+\begin{bmatrix} +2 & -3 \\ 1 & 0 \\ +4 & +8 \end{bmatrix} \quad \begin{bmatrix} 5 & -3 \\ 1 & 0 \\ 10 & 0 \end{bmatrix}$$

$2x + 3 = 9$

$-3 \quad -3$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

$$3X + \begin{bmatrix} 12 & 15 \\ 8 & -1 \end{bmatrix} = \begin{bmatrix} 21 & 10 \\ 23 & 11 \end{bmatrix}$$

$$-\begin{bmatrix} 12 & 15 \\ 8 & -1 \end{bmatrix} \quad \begin{bmatrix} -12 & 15 \\ -8 & +1 \end{bmatrix}$$

$$\frac{1}{3}[3X] = \frac{1}{3}\begin{bmatrix} 19 & 25 \\ 15 & 12 \end{bmatrix} \cdot \frac{1}{3}$$

$$X = \begin{bmatrix} \frac{19}{3} & \frac{25}{3} \\ 5 & 4 \end{bmatrix}$$

Matrix Inverses

$$\begin{bmatrix} 10 & -3 \\ 3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 \\ 3 & -10 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -3 \\ -4 & 2 \end{bmatrix}$$

Not possible

Determine if the matrices are inverses. Why?

$$\begin{bmatrix} 5 & -3 & 2 \\ 8 & 2 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} \frac{4}{73} & \frac{8}{73} & \frac{5}{73} \\ \frac{-17}{73} & \frac{11}{73} & \frac{-3}{73} \\ \frac{-1}{73} & \frac{-3}{146} & \frac{17}{73} \end{bmatrix}$$

3x-y=12 ?

Solve the Matrix Using Inverses

1A. A^{-1} $\begin{bmatrix} 5 & -3 \\ 7 & 8 \end{bmatrix}$ $\cdot$ $\begin{bmatrix} x \\ y \end{bmatrix}$ $=$ $\begin{bmatrix} -20 \\ 33 \end{bmatrix}$

A 2x2 X A 2x1 B

order is important!!

$\begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{A^{-1}B}{\begin{bmatrix} -1 \\ 5 \end{bmatrix}}$

$A \cdot X = B$

2A. $\begin{bmatrix} 15 & -42 \\ 1 & 25 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 396 \\ -57 \end{bmatrix}$

$A^{-1} \cdot A \cdot X = A^{-1}B$

$X =$

Write the **matrix equations** as a **system of equations**.

1B. $5x - 3y = -20$
$7x + 8y = 33$

2B. $15x - 42y = 396$
$1x + 25y = -57$

Write the **system of equations** as **a matrix equation**.

3. $6x + 14y = 53$
$3x - y = 17$

$\begin{bmatrix} 6 & 14 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 53 \\ 17 \end{bmatrix}$

Write as a MATRIX EQUATION & SOLVE

4. $2x + 5y = 6$
$3x - 4y = 9$

$\begin{bmatrix} 2 & 5 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$

$x = 3$
$y = 0$

5. $25x + 41y = -173$
$7x - 35y = 91$

$\begin{bmatrix} 25 & 41 \\ 7 & -35 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -173 \\ 91 \end{bmatrix}$

$x = -2 \quad y = -3$

6. Jake 41 coins in his pocket, consisting of nickels and dimes. The total value of the coins is $3.25. How many of each coin does he have?

$5x + 10y = 325$
$x + y = 41$

$\begin{bmatrix} 5 & 10 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 325 \\ 41 \end{bmatrix}$

$x = 17$ nickels
$y = 24$ dimes