Algebra 2 Notes   Name: Key

Section 8.5– Solving Rational Equations

A rational equation is an equation that contains one or more rational expressions. The time t in hours that it takes to travel d miles can be determined by using the equation $t = \frac{d}{r}$, where r is the average rate of speed. This equation is a rational equation.

To solve a rational equation, start by multiplying each term of the equations by the LCD of all the expressions in the equations. This step eliminates the denominators of the rational expressions and results in an equation you can solve by using algebra.

Example 1: **Solving Rational Equations**. Solve each equation.

a. $\left(\frac{10}{3} = \frac{4}{x} + 2\right) 3x$ $\frac{10}{\cancel{3}} \cdot \frac{\cancel{3}x}{1} = \frac{4}{\cancel{x}} \cdot \frac{3\cancel{x}}{1} + \frac{2}{1} \cdot \frac{3x}{1}$ $10x = 4 + 6x$ $4x = 4$ $\boxed{x = 1}$	b. $\left(x + \frac{8}{x} = 6\right) x$ $x \cdot x + \frac{8}{\cancel{x}} \cdot \frac{\cancel{x}}{1} = 6 \cdot x$ $x^2 + 8 = 6x$ $x^2 - 6x + 8 = 0$ $(x-4)(x-2) = 0$ $\boxed{x = 4 \text{ OR } x = 2}$	c. $\left(x = \frac{6}{x} - 1\right) x$ $x \cdot x = \frac{6}{\cancel{x}} \cdot \frac{\cancel{x}}{1} - 1 \cdot x$ $x^2 = 6 - x$ $x^2 + x - 6 = 0$ $(x+3)(x-2) = 0$ $\boxed{x = -3 \text{ OR } x = 2}$

An extraneous solution is a solution of an equation derived from an original equation that is NOT a solution of the original equation. When you solve a rational equation, it is possible to get extraneous solutions. These values should be eliminated from the solution set. ALWAYS check your solutions by substituting them into the original equation.

Example 2: **Extraneous Solutions**. Solve each equation.

a. $\left(\frac{3x}{x-3} = \frac{2x+3}{x-3}\right) x-3$ $x \neq 3$ $\frac{3x}{\cancel{(x-3)}} \cdot \frac{\cancel{(x-3)}}{1} = \frac{(2x+3)}{\cancel{(x-3)}} \cdot \frac{\cancel{(x-3)}}{1}$ $3x = 2x + 3$ $x = 3$ $\frac{3 \cdot 3}{3-3} \stackrel{?}{=} \frac{2 \cdot 3 + 3}{3-3}$ $\frac{9}{0} = \frac{9}{0}$ yes, but division by 0 is undefined $\boxed{\text{no solution}}$	b. $\frac{16}{x^2-16} = \frac{2}{x-4}$ $\left(\frac{16}{(x-4)(x+4)} = \frac{2}{(x-4)}\right) (x-4)(x+4)$ $x \neq \pm 4$ $\frac{16\cancel{(x-4)}\cancel{(x+4)}}{\cancel{(x-4)}\cancel{(x+4)}} = \frac{2\cancel{(x-4)}(x+4)}{\cancel{(x-4)}}$ $16 = 2x + 8$ $2x = 8$ $\cancel{x = 4}$ $x \neq 4!$ $\boxed{\text{no solution}}$

c. $\left(\frac{2x-9}{x-7}+\frac{x}{2}=\frac{5}{x-7}\right)2(x-7)$ ⟵ $x \neq 7$

$\frac{(2x-9)2\cancel{(x-7)}}{\cancel{(x-7)}}+\frac{\cancel{2}x(x-7)}{\cancel{2}}=\frac{10\cancel{(x-7)}}{\cancel{(x-7)}}$

$4x-18+x^2-7x=10$

$x^2-3x-28=0$

$(x-7)(x+4)=0$

$\cancel{x=7}$ OR $\boxed{x=-4}$

↑ $x \neq 7$!

d. $\left(\frac{1}{x-1}=\frac{x}{x-1}+\frac{x}{6}\right)6(x-1)$ ⟵ $x \neq 1$

$\frac{1\cdot 6\cancel{(x-1)}}{\cancel{(x-1)}}=\frac{6x\cancel{(x-1)}}{\cancel{(x-1)}}+\frac{\cancel{6}x(x-1)}{\cancel{6}}$

$6=6x+x^2-x$

$0=x^2+5x-6$

$0=(x+6)(x-1)$

$\boxed{x=-6}$ OR $\cancel{x=1}$

↑ $x \neq 1$!

e. $\frac{5}{x-2}-\frac{3}{x+3}=\frac{24}{x^2+x-6}$ ⟵ $x \neq -3, 2$

$\left(\frac{5}{x-2}-\frac{3}{x+3}=\frac{24}{(x+3)(x-2)}\right)(x+3)(x-2)$

$\frac{5(x+3)\cancel{(x-2)}}{\cancel{(x-2)}}+\frac{-3\cancel{(x+3)}(x-2)}{\cancel{(x+3)}}=\frac{24\cancel{(x+3)}\cancel{(x-2)}}{\cancel{(x+3)}\cancel{(x-2)}}$

$5x+15-3x+6=24$

$2x+21=24$

$2x=3$

$\boxed{x=\frac{3}{2}}$

f. $\left(\frac{8}{x+2}+4=\frac{4x}{x-3}\right)(x+2)(x-3)$ ⟵ $x \neq -2, 3$

$\frac{8\cancel{(x+2)}(x-3)}{\cancel{(x+2)}}+\frac{4(x+2)(x-3)}{1}=\frac{4x(x+2)\cancel{(x-3)}}{\cancel{(x-3)}}$

$8x-24+4(x^2-x-6)=4x^2+8x$

$8x-24+4x^2-4x-24=4x^2+8x$

$4x^2+4x-48=4x^2+8x$

$4x-48=8x$

$-48=4x$

$\boxed{x=-12}$

Always check your solutions by plugging them back into the ORIGINAL equation. You may be sorry if you don't...