Algebra 2 Notes

Name: Key

Section 8.1– Variation Functions

In Chapter 2, you studied many types of linear functions. Once special type of linear function is called a direct variation. A direct variation is a relationship between two variables x and y that can be written in the form $y = kx$, where $k \neq 0$. In this relationship, k is the constant of variation. For the equation $y = kx$, y varies directly as x.

A direct variation equation is a linear equation in the form $y = mx + b$, where $b = 0$ and the constant of variation k is the slope. Because $b = 0$, the graph of a direct variation always passes through the origin.

Example 1: **Writing and Graphing Direct Variation**

a. Given: y varies directly as x, and $y = 14$ when $x = 3.5$. Write and graph the direct variation function.

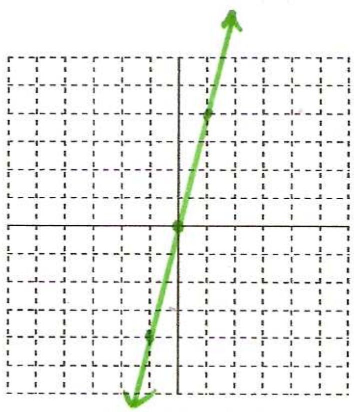
$y = kx$

$14 = k(3.5)$

$k = \frac{14}{3.5}$

$k = 4$

$\boxed{y = 4x}$



b. Given: y varies directly as x, and $y = 6.5$ when $x = 13$. Write and graph the direct variation function.

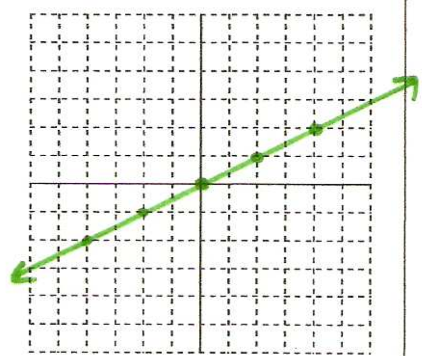
$y = kx$

$6.5 = k(13)$

$k = \frac{6.5}{13}$

$k = \frac{1}{2}$

$\boxed{y = \frac{1}{2}x}$



Example 2: **Solving Direct Variation Problems**

a. The circumference of a circle C varies directly as the radius r, and $C = 7\pi$ ft when $r = 3.5$ ft. Find r when $C = 4.5\pi$ ft.

$C = kr$

$7\pi = k(3.5)$

$k = \frac{7\pi}{3.5}$

$k = 2\pi$

So, $C = 2\pi r$

$4.5\pi = 2\pi r$

$r = \frac{4.5\pi}{2\pi}$

$r = 2.25$

$\boxed{2.25 \text{ ft}}$

b. The perimeter P of a regular dodecagon varies directly as the side length s, and $P = 18$ in when $s = 1.5$ in. Find s when $P = 75$ in.

$P = ks$

$18 = k(1.5)$

$k = \frac{18}{1.5}$

$k = 12$

So, $P = 12s$

$75 = 12s$

$s = \frac{75}{12}$

$s = 6.25$

$\boxed{6.25 \text{ in}}$

A joint variation is a relationship among 3 variables that can be written in the form $y = kxz$, where k is the constant of variation. For the equation $y = kxz$, y varies jointly as x and z.

Example 3: Solving Joint Variation Problems.

The area A of a triangle varies jointly as the base b and height h, and $A = 12\ \text{m}^2$ when $b = 6$ m and $h = 4$ m. Find b when $A = 36\ \text{m}^2$ and $h = 8$ m.

$A = kbh$

$12 = k \cdot 6 \cdot 4$

$12 = k \cdot 24$

$k = \frac{12}{24} \longrightarrow k = \frac{1}{2}$

So, $A = \frac{1}{2}bh$

$36 = \frac{1}{2} b \cdot 8$

$36 = 4b$

$b = 9$

$\boxed{9\ \text{m}}$

A third type of variation describes a situation in which one quantity increases and the other decreases. For example, the table shows that the time needed to drive 600 miles decreases as speed increases.

Speed (mi/h)	Time (h)	Distance (mi)
30	20	600
40	15	600
50	12	600

This type of variation is an inverse variation. An inverse variation is a relationship between two variables x and y that can be written in the form $y = \frac{k}{x}$, where $k \neq 0$. For the equation $y = \frac{k}{x}$, y varies inversely as x.

Example 1: ***Writing and Graphing Inverse Variation***

a. Given: y varies inversely as x, and $y = 3$ when $x = 2$. Write and graph the direct variation function.

$y = \frac{k}{x} \longleftarrow x \neq 0!$

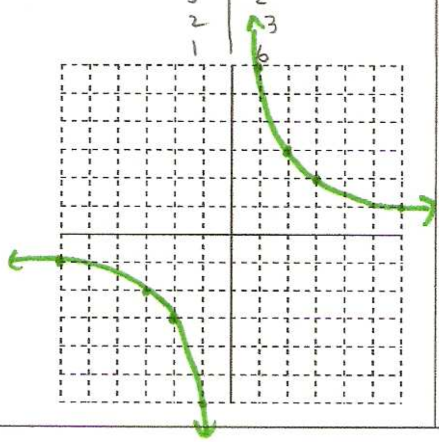
$3 = \frac{k}{2}$

$k = 3 \cdot 2$

$k = 6$

$\boxed{y = \frac{6}{x}}$

x	y
-2	-3
-3	-2
-1	-6
3	2
2	3
1	6



b. Given: y varies inversely as x, and $y = +2$ when $x = 5$. Write and graph the direct variation function.

$y = \frac{k}{x}$

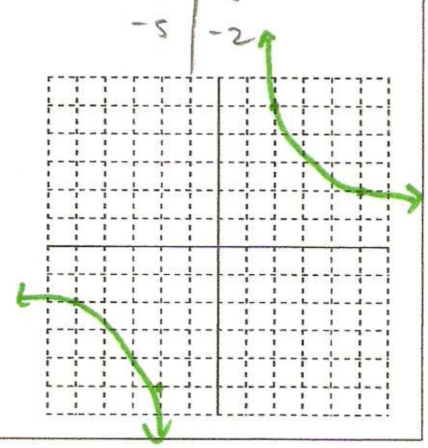
$2 = \frac{k}{5}$

$k = 2 \cdot 5$

$k = 10$

$\boxed{y = \frac{10}{x}}$

x	y
1	10
2	5
5	2
-2	-5
-5	-2



Example 5: **Community Service Application**

The time t that it takes for a group of volunteers to construct a house varies inversely as the number of volunteers v. If 20 volunteers can build a house in 62.5 working hours, how many volunteers would be needed to build a house in 50 working hours?

$t = \frac{k}{v}$

$62.5 = \frac{k}{20}$

$k = 20(62.5)$

$k = 1250$

so, $t = \frac{1250}{v}$

$50 = \frac{1250}{v}$

$50v = 1250$

$v = \frac{1250}{50} \longrightarrow 25$

$\boxed{\text{25 volunteers}}$

You can use algebra to rewrite variation functions in terms of k.

Direct Variation	Inverse Variation
$y = kx \rightarrow$ $k = \frac{y}{x}$	$y = \frac{k}{x} \rightarrow$ $k = xy$

Notice that in direct variation, the ______________ of the two quantities is constant. In inverse variation, the ______________ of the two quantities is constant.

Example 6: **Identifying Direct and Inverse Variations:** Determine whether each data set represents a direct variation, an inverse variation, or neither. Justify your answer.

a.

x	3	8	10
y	9	24	30

$\frac{y}{x}$: $\frac{9}{3} = \frac{24}{8} = \frac{30}{10}$ ✓

xy: $27 \neq 192 \neq 300$

direct

b.

x	4.5	12	2
y	8	3	18

$\frac{y}{x}$: $\frac{8}{4.5} \neq \frac{3}{12} \neq \frac{18}{2}$

xy: $36 = 36 = 36$ ✓

inverse

A combined variation is a relationship that contains both direct and inverse variation. Quantities that vary directly appear in the numerator, and quantities that vary inversely appear in the denominator.

Example 7: **Chemistry Application.**

The volume V of a gas varies inversely as the pressure P and directly as the temperature T. A certain gas has a volume of 10 liters (L), a temperature of 300 kelvins (K), and a pressure of 1.5 atmospheres (atm). If the gas is compressed to a volume of 7.5 L and is heated to 350 K, what will the new pressure be?

$V = \frac{kT}{P}$

$10 = \frac{k \cdot 300}{1.5}$

$10 = 200k$

$k = 0.05$

so, $V = \frac{0.05T}{P}$

$7.5 = \frac{0.05(350)}{P}$

$7.5P = 17.5$

$P = 2\frac{1}{3}$

$\boxed{2\frac{1}{3} \text{ atm}}$