Algebra 2 Notes Name: key

Section 8.2– Multiplying and Dividing Rational Expressions

A rational expression is a quotient of two polynomials. Some examples and some NON-examples of rational expressions can be seen below:

Examples: $\frac{x^2-4}{x+2}$ $\frac{10}{x^2-6}$ $\frac{x+3}{x-7}$ $\frac{x^2-5x+6}{x^2+3x+2}$ $\frac{12x^2}{y^5}$

NON-Example: $\frac{x+2}{3}$ $\frac{2}{5}$ x^3-1 $(x+3)(x-2)$ $\frac{x^4+2}{7}$

Because rational expressions are ratios of polynomials, you can simplify them the same way as you simplify fractions. Recall that to write a fraction in simplest form, you can divide out common factors in the numerator and denominator.

Let's see what we mean by simplifying a basic fraction: $\frac{9}{24} = \frac{\cancel{3}\cdot 3}{\cancel{3}\cdot 8} = \boxed{\frac{3}{8}}$

Example 1: Simplify. Be sure to identify **ANY** x-values for which the **ORIGINAL** expression is undefined. **HINT:** Think about when the denominator would be equal to zero.

a.	b.	c.
$\frac{3x^7}{2x^4} = \frac{3x^{7-4}}{2} = \boxed{\frac{3x^3}{2}}$	$\frac{16x^{11}}{8x^2} = \frac{2\cdot\cancel{8}x^{11-2}}{\cancel{8}} = \boxed{2x^9}$	$\frac{12x^1}{16x^5} = \frac{3\cdot\cancel{4}}{4\cdot\cancel{4}x^{5-1}} = \boxed{\frac{3}{4x^4}}$
$x \neq 0$	$x \neq 0$	$x \neq 0$

The previous examples could be simplified quickly because the numerator and denominator of each rational expression were monomials. Life is not always so simple. ☹ But, never fear! When you are faced with a numerator and/or denominator that are binomials or trinomials, you just need to factor first before you can simplify. To show WHY you would need to do this, consider the following...

If given $\frac{2+4}{2}$, would you cross out the 2s and get a result of 4? NO!

Hopefully you said **NO**! How would you do this problem?

$$\frac{2+4}{2} = \frac{6}{2} = \frac{\cancel{2}\cdot 3}{\cancel{2}} = 3$$

We have to use the same idea when dealing with polynomials. FACTOR FIRST! And then get to canceling!

Example 2: Simplify. Be sure to identify **ANY** x-values for which the **ORIGINAL** expression is undefined.

a. $\dfrac{x-5}{x^2-25} = \dfrac{\cancel{(x-5)}}{\cancel{(x-5)}(x+5)}$

$= \boxed{\dfrac{1}{x+5}}$

$x-5 \neq 0$ and $x+5 \neq 0$

$x \neq 5$ $\quad x \neq -5$

$x \neq \pm 5$

b. $\dfrac{x^2-2x-3}{x^2+5x+4} = \dfrac{(x-3)\cancel{(x+1)}}{(x+4)\cancel{(x+1)}}$

$= \boxed{\dfrac{x-3}{x+4}}$

$x+4 \neq 0$ & $x+1 \neq 0$

$x \neq -4,\ x \neq -1$

c. $\dfrac{3x+4}{3x^2+x-4} = \dfrac{\cancel{(3x+4)}}{\cancel{(3x+4)}(x-1)}$

$= \boxed{\dfrac{1}{x-1}}$

$3x+4 \neq 0$ & $x-1 \neq 0$

$3x \neq -4$

$x \neq -\dfrac{4}{3},\ x \neq 1$

d. $\dfrac{32-2x^2}{x^2-x-12} = \dfrac{-2(x^2-16)}{(x-4)(x+3)}$

$= \dfrac{-2(x+4)\cancel{(x-4)}}{\cancel{(x-4)}(x+3)}$

$= \boxed{\dfrac{-2(x+4)}{x+3}}$

$x \neq 4,\ x \neq -3$

You can multiply rational expressions the same way that you multiply fractions!

Let's see what we mean by multiplying with basic fractions: $\dfrac{2}{5}\cdot\dfrac{15}{8} = \dfrac{\cancel{2}}{\cancel{5}}\cdot\dfrac{\cancel{5}\cdot 3}{\cancel{2}\cdot 4} = \boxed{\dfrac{3}{4}}$

Before we start multiplying rational expressions, let's go over the "STEPS" in the table below.

Multiplying Rational Expressions
1. Factor out all numerators and denominators COMPLETELY.
2. Divide out common factors of the numerators and denominators.
3. Multiply numerators. Then multiply denominators.
5. Be sure the numerator and denominator have no common factors other than 1.

Example 3: Multiply. Assume that all expressions are defined.

a. $\dfrac{2x^4y^5}{3x^2}\cdot\dfrac{15x^2}{8x^3y^2} = \dfrac{\cancel{2}\cdot\cancel{3}\cdot 5\cdot x^{4+2}\, y^5}{\cancel{3}\cdot\cancel{2}\cdot 4\cdot x^{2+3}\, y^2}$

$= \dfrac{5x^6y^5}{4x^5y^2}$

$= \dfrac{5x^{6-5}\cdot y^{5-2}}{4\cdot}$

$= \boxed{\dfrac{5xy^3}{4}}$

b. $\dfrac{x}{15}\cdot\dfrac{x^7}{2x}\cdot\dfrac{20}{x^4} = \dfrac{\cancel{2}\cdot 10\cdot x^{1+7}}{\cancel{2}\cdot 15\cdot x^{1+4}}$

$= \dfrac{2\cdot\cancel{5}\, x^8}{3\cdot\cancel{5}\, x^5}$

$= \dfrac{2x^{8-5}}{3}$

$= \boxed{\dfrac{2x^3}{3}}$

Example 4: Multiply. Assume that all expressions are defined.

a. $\dfrac{x+2}{3x+12}\cdot\dfrac{x+4}{x^2-4} = \dfrac{\cancel{(x+2)}}{3\cancel{(x+4)}}\cdot\dfrac{\cancel{(x+4)}}{(x-2)\cancel{(x+2)}}$

$= \boxed{\dfrac{1}{3(x-2)}}$

b. $\dfrac{40-10x}{x^2-6x+8}\cdot\dfrac{x+3}{5x+15}$

$= \dfrac{-\overset{2}{\cancel{10}}\cancel{(x-4)}}{\cancel{(x-4)}(x-2)}\cdot\dfrac{\cancel{(x+3)}}{\underset{1}{\cancel{5}}\cancel{(x+3)}}$

$= \boxed{\dfrac{-2}{x-2}}$

You can also divide rational expressions. Recall that to divide by a fraction, you multiply by the reciprocal.

Let's see what we mean by multiplying with basic fractions: $\dfrac{3}{4}\div\dfrac{15}{8} = \dfrac{3}{4}\cdot\dfrac{8}{15}$

$= \dfrac{\cancel{3}}{\cancel{4}}\cdot\dfrac{2\cdot\cancel{4}}{\cancel{3}\cdot 5}$

$= \boxed{\dfrac{2}{5}}$

Example 5: ~~Multiply~~ Divide. Assume that all expressions are defined.

a. $\dfrac{4x^3}{9x^2y} \div \dfrac{16}{9y^5} = \dfrac{\overset{1}{\cancel{4}}x^3}{\cancel{9}x^2y} \cdot \dfrac{\cancel{9}y^5}{\underset{4}{\cancel{16}}}$

$= \dfrac{x^3y^5}{4x^2y}$

$= \dfrac{x^{3-2} \cdot y^{5-1}}{4}$

$= \boxed{\dfrac{xy^4}{4}}$

b. $\dfrac{x^2}{4} \div \dfrac{x^4y}{12y^2} = \dfrac{x^2}{\underset{1}{\cancel{4}}} \cdot \dfrac{\overset{3}{\cancel{12}}y^2}{x^4y}$

$= \dfrac{3x^2y^2}{x^4y}$

$= \dfrac{3y^{2-1}}{x^{4-2}}$

$= \boxed{\dfrac{3y}{x^2}}$

Example 6: ~~Multiply~~ Divide. Assume that all expressions are defined.

a. $\dfrac{2x^2-7x-4}{x^2-9} \div \dfrac{4x^2-1}{2x^2-7x+3}$

$\dfrac{2x^2-7x-4}{x^2-9} \cdot \dfrac{2x^2-7x+3}{4x^2-1}$

$\dfrac{\cancel{(2x+1)}(x-4)}{(x+3)\cancel{(x-3)}} \cdot \dfrac{\cancel{(x-3)}\cancel{(2x-1)}}{\cancel{(2x+1)}\cancel{(2x-1)}}$

$\boxed{\dfrac{x-4}{x+3}}$

b. $\dfrac{x^5-4x^3}{x^2-x-2} \div \dfrac{x^5-x^4-2x^3}{x^2-1}$

$\dfrac{x^5-4x^3}{x^2-x-2} \cdot \dfrac{x^2-1}{x^5-x^4-2x^3}$

$\dfrac{x^3(x^2-4)}{(x-2)(x+1)} \cdot \dfrac{(x+1)(x-1)}{x^3(x^2-x-2)}$

$\dfrac{\cancel{x^3}(x+2)\cancel{(x-2)}}{(x-2)(x+1)} \cdot \dfrac{\cancel{(x+1)}(x-1)}{\cancel{x^3}\cancel{(x-2)}\cancel{(x+1)}}$

$\boxed{\dfrac{(x+2)(x-1)}{(x-2)(x+1)}}$

It will be important for you to (a) know how to factor, and (b) to practice by doing the homework!