Algebra 2 Notes Name: Key

Section 7.5 - Properties of Logarithms

Because logarithms are exponents, you can derive the properties of logarithms from the properties of exponents.

Remember that to *multiply* powers with the same base, you *add* exponents. $b^m b^n =$ b^{m+n}

Product Property of Logarithms		
For any positive numbers m, n, and b $(b \neq 1)$,		
Words	**Numbers**	**Algebra**
The logarithm of a product is equal to the sum of the logarithms of its factors.	$\log_3 1000 = \log_3 (10 \cdot 100)$ $= \log_3 10 + \log_3 100$	$\log_b mn = \log_b m + \log_b n$

The property above can be used in reverse to write a sum of logarithms (exponents) as a single logarithm, which can often be simplified.

Example 1: Express as a single logarithm. Simplify, if possible.

a. $\log_4 2 + \log_4 32$ $\log_4 (2 \cdot 32)$ $\log_4 64$ 4 to what power is 64? $\boxed{3}$	b. $\log_{\frac{1}{3}} 27 + \log_{\frac{1}{3}} \frac{1}{9}$ $\log_{\frac{1}{3}} (27 \cdot \frac{1}{9})$ $\log_{\frac{1}{3}} 3$ $\frac{1}{3}$ to what power is 3? $\boxed{-1}$

Remember that to *divide* powers with the same base, you *subtract* exponents. $\frac{b^m}{b^n} =$ _____

Because logarithms are exponents, subtracting logarithms with the same base is the same as finding the logarithm of the quotient with that base.

Quotient Property of Logarithms		
For any positive numbers m, n, and b $(b \neq 1)$,		
Words	**Numbers**	**Algebra**
The logarithm of a quotient is logarithm of the dividend minus the logarithm of the divisor.	$\log_5 \left(\frac{16}{2}\right) = \log_5 16 - \log_5 2$	$\log_b \frac{m}{n} = \log_b m - \log_b n$

Example 2: Express as a single logarithm. Simplify, if possible.

a. $\log_2 32 - \log_2 4$	b. $\log_7 49 - \log_7 7$
$\log_2 \frac{32}{4}$ $\log_2 8$ 2 to what power is 8? $\boxed{3}$	$\log_7 \frac{49}{7}$ $\log_7 7$ 7 to what power is 7? $\boxed{1}$

Because you can multiply logarithms, you can also take powers of logarithms.

Power Property of Logarithms		
For any real number p and positive numbers a and b $(b \neq 1)$,		
Words	**Numbers**	**Algebra**
The logarithm of a power is the product of the exponent and the logarithm of the base.	$\log 10^3$ $= \log(10 \cdot 10 \cdot 10)$ $= \log 10 + \log 10 + \log 10$ $= 3\log 10$	$\log_b a^p = p\log_b a$

Example 3: Express as a product. Simplify, if possible.

a. $\log_3 81^2$	b. $\log_2 \left(\frac{1}{2}\right)^5$
$2\log_3 81$ $2 \cdot 4$ $\boxed{8}$	$5\log_2\left(\frac{1}{2}\right)$ $5 \cdot -1$ $\boxed{-5}$

Exponential and logarithmic operations undo each other since they are inverse operations.

Inverse Properties of Logarithms and Exponents	
For any base b such that > 0 and $b \neq 1$,	
Algebra	**Example**
$\log_b b^x = x$	$\log_5 5^2 = 2$
$b^{\log_b x} = x$	$8^{\log_8 9} = 9$

Example 4: Simplify each expression.

a. $\log_3 3^{2x-7}$	b. $\log_5 125$	c. $2^{\log_2 27}$
$(2x-7)\log_3 3$ $(2x-7) \cdot 1$ $\boxed{2x-7}$	$\log_5 5^3$ $3 \cdot \log_5 5$ $3 \cdot 1$ $\boxed{3}$	$\boxed{27}$

Most calculators calculate logarithms only in base 10 or base e (Section 7.6). You can change a logarithm in one base to a logarithm in another base with the following formula...

Change of Base Formula	
For $a > 0$ and $a \neq 1$ and any base b such that $b > 0$ and $b \neq 1$,	
Algebra	**Example**
$\log_b x = \dfrac{\log_a x}{\log_a b}$	$\log_4 8 = \dfrac{\log_2 8}{\log_2 4}$

Example 5: Evaluate each logarithm. Round to the nearest hundredth when necessary.

a. $\log_4 8$	b. $\log_{50} 43$
$\dfrac{\log 8}{\log 4}$ Used base 10 for calc.	$\dfrac{\log 43}{\log 50}$
$\boxed{1.5}$	$\boxed{\approx .96}$

Example 6: Geology Application.

Seismologists use the Richter scale to express the energy, or magnitude, of an earthquake. The Richter magnitude of an earthquake, M, is related to the energy released in ergs E shown by the formula $M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$. Hoe many times as much energy is released by an earthquake with a magnitude of 9.2 than by an earthquake with a magnitude of 8?

$9.2 = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$

$13.8 = \log\left(\frac{E}{10^{11.8}}\right)$

$10^{13.8} = \frac{E}{10^{11.8}}$

$E = 10^{13.8} \cdot 10^{11.8}$

$E = 10^{25.6}$

$8 = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$

$12 = \log\left(\frac{E}{10^{11.8}}\right)$

$10^{12} = \frac{E}{10^{11.8}}$

$E = 10^{12} \cdot 10^{11.8}$

$E = 10^{23.8}$

$\frac{10^{25.6}}{10^{23.8}}$ $\boxed{\approx 63 \text{ times}}$

You need to be sure to memorize the properties of logarithms and the change-of-base formula. These are key concepts we will need to use when later solving equations with logarithms.