Algebra 2 Notes Name: ________________
Section 7.3 – Logarithmic Functions

Question 1: How many times would you have to double \$1 before you had \$8?

$1 \cdot 2 = 2$
$2 \cdot 2 = 4$
$4 \cdot 2 = 8$

3 times

$1 \cdot 2^x = 8$
$2^x = 8$
$x = 3$

Question 2: How many times would you have to double \$1 before you had \$2,097,152?

Too much time!

$1 \cdot 2^x = 2097152$

We could answer Question 2 a lot faster if we had an inverse operation that could undo raising a base to an exponent. This operation is called finding the logarithm. A logarithm is the exponent to which a specified base is raised to obtain a given value.

You can write an exponential equation as a logarithmic equation and vice versa.

Exponential Function	Logarithmic Function
$b^x = a$ — exponent; base; "equals"	$\log_b a = x$ — exponent; base; argument

Example 1: Write each exponential equation in logarithmic form.

a. $2^6 = 64$	b. $1 = 5^0$	c. $5^{-2} = 0.04$	d. $3^x = 81$
$\log_2 64 = 2$	$\log_5 1 = 0$	$\log_5 0.04 = -2$	$\log_3 81 = x$

Example 2: Write each logarithmic equation in exponential form.

a. $\log_7 49 = 2$	b. $\log_8 0.125 = -1$	c. $3 = \log_5 125$	d. $x = \log_6 \frac{1}{216}$
$7^2 = 49$	$8^{-1} = 0.125$	$5^3 = 125$	$6^x = \frac{1}{216}$

A logarithm is an exponent, so the rules for exponents also apply to logarithms.

Special Properties of Logarithms		
For any base b such that $b > 0$ and $b \neq 1$,		
Logarithmic Form	**Exponential Form**	**Example**
Logarithm of Base b $\log_b b = 1$	$b^1 = b$	$\log_4 4 =$ 1
Logarithm of 1 $\log_b 1 = 0$	$b^0 = 1$	$\log_9 1 =$ 0

A logarithm with base 10 is called a Common logarithm. If no base is written for a logarithm, the base is assumed to be 10. For example, $\log 5 =$ $\log_{10} 5$.

You can use mental math to evaluate some logarithms.

Example 3: Evaluate WITHOUT the calculator.

a. $\log 1000$	b. $\log_4 \frac{1}{4}$	c. $\log_2 16$	d. $\log_{\frac{1}{3}} 27$
10 to what power equals 1000?	4 to what power equals $\frac{1}{4}$?	2 to what power equals 16?	$\frac{1}{3}$ to what power equals 27?
$\boxed{3}$	$\boxed{-1}$	$\boxed{4}$	$\boxed{-3}$

Because logarithms are inverses of exponents, the inverse of an exponential function, such as $y = 2^x$, is a logarithmic function $y = \log_2 x$.

Example 4: Use the given x-values to graph each function. Then graph its inverse. Describe the domain and range of the function and its inverse function.

a. $f(x) = 2^x$; $x = -2, -1, 0, 1,$ ~~and~~ 2, and 3

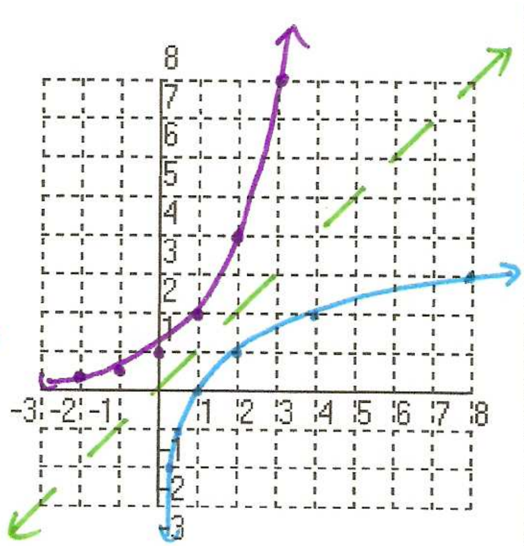
x	$f(x)$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

$f(x)$
D: $\mathbb{R}$
R: $y > 0$

$f^{-1}(x) = \log_2 x$

x	$f^{-1}(x)$
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3

$f^{-1}(x)$
D: $x > 0$
R: $\mathbb{R}$



b. $f(x) = 0.8^x$; $x = -3,\ 0,\ 1,\ 4,$ and 7

x	$f(x)$
-3	≈ 2
0	1
1	0.8
4	$\approx .4$
7	$\approx .2$

$f^{-1}(x) = \log_{0.8} x$

x	$f^{-1}(x)$
2	-3
1	0
.8	1
.4	4
.2	7

$f(x)$

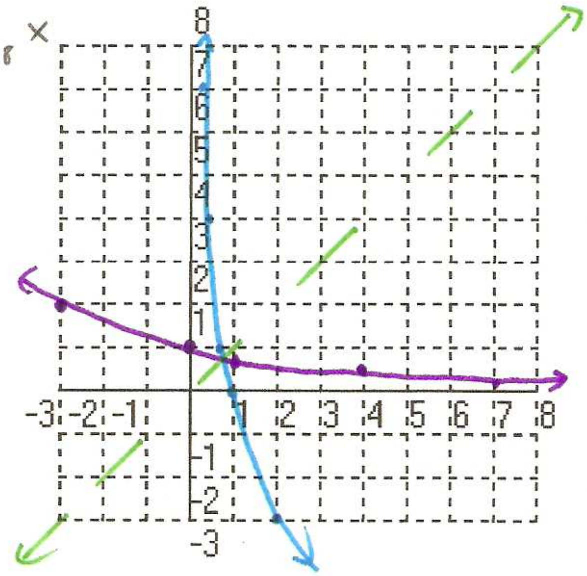
D: $\mathbb{R}$

R: $y>0$

$f^{-1}(x)$

D: $x>0$

R: $\mathbb{R}$



Example 5: Environmental Application

Chemists regularly test rain samples to determine the rain's acidity, or concentration of hydrogen ions (H^+). Acidity is measured in pH, as given by the function $pH = -\log[H^+]$, where $[H^+]$ represents the hydrogen ion concentration in moles per liter.

Find the pH of rainwater from each location.

(a) Central New Jersey, given that the hydrogen ion concentration is 0.0000316 moles per liter.

$pH = -\log[0.0000316]$

$\boxed{pH \approx 4.5}$

(b) Central North Dakota, given that the hydrogen ion concentration is 0.0000009 moles per liter.

$pH = -\log[0.0000009]$

$\boxed{pH \approx 6.0}$