

Algebra 2 Notes

Name: key

Section 7.5 - Exponential and Logarithmic Functions

DAY ONE:

An exponential equation is an equation containing one or more expressions that have a variable as an exponent. To solve exponential equations today, we will try writing them so that the bases are the same...

If $b^x = b^y$, then $x = y$. ($b \neq 0, b \neq 1$)

Example 1: Solve and check.

<p>a. $25^{x-2} = 5$ $(5^2)^{x-2} = 5$ $5^{2(x-2)} = 5^1$ $5^{2x-4} = 5^1$ $2x-4 = 1$ $2x = 5$ $x = \frac{5}{2}$</p>	<p>b. $8^x = 2^{x+6}$ $(2^3)^x = 2^{x+6}$ $2^{3x} = 2^{x+6}$ $3x = x+6$ $2x = 6$ $x = 3$</p>
<p>c. $27^{\frac{x}{2}} = \frac{1}{9}$ $(3^3)^{\frac{x}{2}} = 3^{-2}$ $3^{\frac{3x}{2}} = 3^{-2}$ $\frac{3x}{2} = -2$ $3x = -4$ $x = -\frac{4}{3}$</p>	<p>d. $\left(\frac{1}{3}\right)^{2x-3} = 27^x$ $(3^{-1})^{2x-3} = (3^{+3})^x$ $3^{-(2x-3)} = 3^{3x}$ $3^{-2x+3} = 3^{3x}$ $-2x+3 = 3x$ $3 = 5x$ $x = \frac{3}{5}$</p>

A logarithmic equation is an equation with a logarithmic expression that contains a variable. You can solve logarithmic equations by using the properties of logarithms we learned in Section 7.4.

$\log_b x = y$ equivalent to $b^y = x$. AND If $\log_b x = \log_b y$, then $x = y$.

Example 2: Solve and check.

<p>a. $\log_5(x-5) = 2$ $5^2 = x-5$ $25 = x-5$ $x = 30$</p>	<p>b. $\log(45x) - \log 3 = 1$ $\log \frac{45x}{3} = 1$ $\log(15x) = 1$ $10^1 = 15x$ $x = \frac{10}{15}$ $x = \frac{2}{3}$</p>
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c. $2\log_4 x = 7$

$$\log_4 x = \frac{7}{2} \quad \text{OR} \quad \log_4 x^2 = 7$$

$$4^{\frac{7}{2}} = x$$

$$x = 2^7$$

$$\boxed{x = 128}$$

$$4^7 = x^2$$

$$\sqrt{x^2} = \sqrt{16384}$$

$$x = \pm 128$$

$$\boxed{x = 128}$$

d. $\log(x+2) = 3 + \log 8$

$$\log(x+2) - \log 8 = 3$$

$$\log \frac{x+2}{8} = 3$$

$$10^3 = \frac{x+2}{8}$$

$$1000 = \frac{x+2}{8}$$

$$8000 = x+2$$

$$\boxed{x = 7998}$$

can't take log of a negative!
Tomorrow, we will need to be able to use the change-of-base formula. Let's review it now...

Example 3: Use the change of base formula to evaluate each logarithm to the nearest hundredth.

a. $\log_7 18$

$$\frac{\log 18}{\log 7}$$

$$\boxed{\approx 1.49}$$

b. $\log_{52} 109$

$$\frac{\log 109}{\log 52}$$

$$\boxed{\approx 1.19}$$

c. $\log_{40} 5$

$$\frac{\log 5}{\log 40}$$

$$\boxed{\approx 0.44}$$

DAY TWO:

Let's take some of the problems from yesterday and make them a bit more challenging. ☺

Example 4: Solve and check.

a. $\log x + \log(x+9) = 1$

$$\log x(x+9) = 1$$

$$10^1 = x(x+9)$$

$$10 = x^2 + 9x$$

$$0 = x^2 + 9x - 10$$

$$0 = (x+10)(x-1)$$

$$x+10=0 \quad x-1=0$$

$$\cancel{x = -10}$$

$$\boxed{x = 1}$$

b. $\log_2(x-4) + \log_2 x = 5$

$$\log_2 x(x-4) = 5$$

$$2^5 = x(x-4)$$

$$32 = x^2 - 4x$$

$$0 = x^2 - 4x - 32$$

$$0 = (x-8)(x+4)$$

$$x-8=0$$

$$x+4=0$$

$$\boxed{x = 8}$$

$$\cancel{x = -4}$$

Sometimes, we will solve exponential equations where it is not easy to get the same base. When this is the case, we can either take the log of both sides OR we can rewrite the equation in logarithmic form. It will be up to you which method to use, but we will show both ways in the examples.

Example 5: Solve and check. Round to the nearest hundredth.

a. $5^{x-2} = 200$

$$\log_5 200 = x - 2$$

$$x = \log_5 200 + 2$$

$$x = \frac{\log 200}{\log 5} + 2$$

$$x \approx 5.29$$

OR $\log_5 5^{x-2} = \log_5 200$

$$(x-2)\log 5 = \log 200$$

$$x-2 = \frac{\log 200}{\log 5}$$

$$x = \frac{\log 200}{\log 5} + 2$$

$$x \approx 5.29$$



Same answer, different methods!

b. $10^{3x} + 6 = 25$

$$10^{3x} = 19$$

$$\log_{10} 19 = 3x$$

$$\log 19 = 3x$$

$$x = \frac{\log 19}{3}$$

$$x \approx 0.43$$

OR $\log 10^{3x} = \log 19$

$$3x \log 10 = \log 19$$

$$x = \frac{\log 19}{3 \log 10}$$

$$x \approx 0.43$$



c. $62^{\frac{x}{2}} - 12 = 4$

$$62^{\frac{x}{2}} = 16$$

$$\log_{62} 16 = \frac{x}{2}$$

$$x = 2 \log_{62} 16$$

$$x = 2 \left(\frac{\log 16}{\log 62} \right)$$

$$x \approx 1.34$$

I like this method best.

d. $2^{3x+1} = 34$

Maybe you like it better this way...

$$\log 2^{3x+1} = \log 34$$

$$(3x+1) \log 2 = \log 34$$

$$3x+1 = \frac{\log 34}{\log 2}$$

$$3x = \frac{\log 34}{\log 2} - 1$$

$$x = \frac{\frac{\log 34}{\log 2} - 1}{3}$$

$$x \approx 1.36$$

You will need to be comfortable with how to solve both exponential and logarithmic equations. On a future assignment, as well as on the test, the types of equations will be mixed. You will need to know what method to use in order to solve the equation. Also, be sure you are checking your answers by plugging the x-value back into your original equation. ☺