Algebra 2 Notes Name: Key

Section 7.2 – Inverses of Relations and Functions

You have seen the word inverse used in various ways:

- The additive inverse of 3 is -3.
- The multiplicative inverse of 5 is 1/5.
- The multiplicative inverse matrix of $A=\begin{bmatrix}3 & 1\\ 4 & 2\end{bmatrix}$ is $A^{-1}=\begin{bmatrix}1 & -0.5\\ -2 & 1.5\end{bmatrix}$.

You can also find and apply inverses to relations and functions. To graph the inverse relation, you can reflect each point across the line $y=x$. This is equivalent to switching the x- and y-values in each ordered pair or relation.

Example 1: Graph the relation and connect the points. Then graph the inverse. Identify the domain and range of each relation.

Relation:

x	0	1	2	4	8
y	2	4	5	6	7

Domain: $0 \le x \le 8$

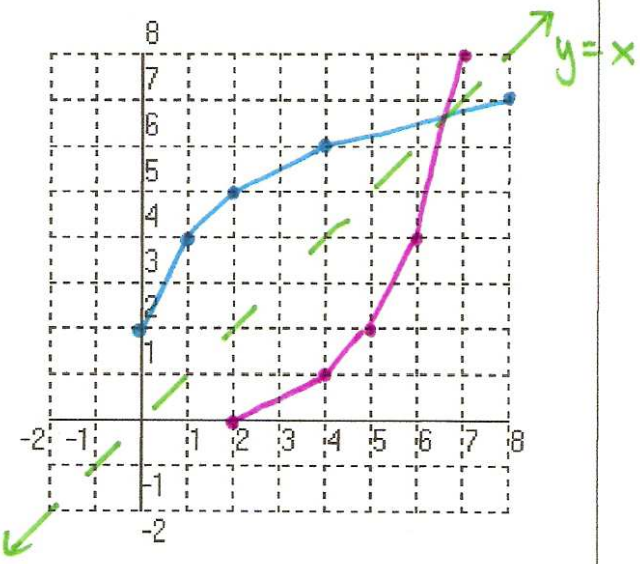
Range: $2 \le y \le 7$

Inverse:

x	2	4	5	6	7
y	0	1	2	4	8

Domain: $2 \le x \le 7$

Range: $0 \le y \le 8$



When the relation is also a function, you can write the inverse of the function $f(x)$ as $f^{-1}(x)$. This notation does **NOT** indicate a reciprocal.

Functions that undo each other are inverse functions.

Input	→	Function	→	Output
3		$f(x)=x+6$		9

Input	→	Function	→	Output
9		$f^{-1}(x)=x-6$		3

To find the inverse function, use the inverse operation. Another option would be to switch the x and y in the equation for the original function, and then solve for y.

Example 2: Given $f(x)$, find the equation of its inverse, $f^{-1}(x)$.

a. $f(x) = 2x$

$y = 2x$

inverse

$x = 2y$

$y = \frac{x}{2}$

$f^{-1}(x) = \frac{x}{2}$

b. $f(x) = \frac{x}{4} - 5$

$y = \frac{x}{4} - 5$

inverse

$x = \frac{y}{4} - 5$

$\frac{y}{4} = x + 5$

$y = 4(x+5)$

$f^{-1}(x) = 4x + 20$

c. $f(x) = 5x + 7$

$y = 5x + 7$

inverse

$x = 5y + 7$

$\frac{5y}{5} = \frac{x-7}{5}$

$f^{-1}(x) = \frac{x-7}{5}$

d. $f(x) = \frac{2}{3}x - 8$

$y = \frac{2}{3}x - 8$

inverse

$x = \frac{2}{3}y - 8$

$\frac{3}{2}\left(\frac{2}{3}y\right) = (x+8)\frac{3}{2}$

$y = \frac{3}{2}x + 12$

$f^{-1}(x) = \frac{3}{2}x + 12$

Example 3: Graph $f(x) = 3x + 6$. Then write and graph its inverse.

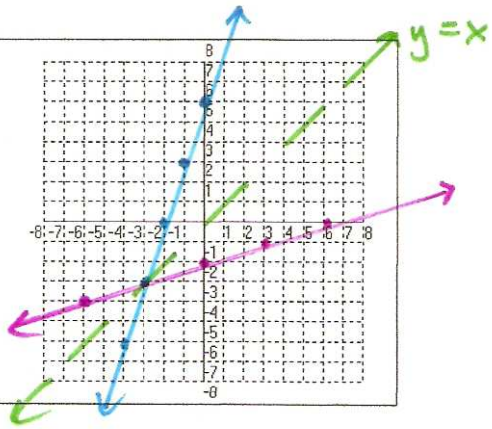
$y = 3x + 6$

inverse

$x = 3y + 6$

$\frac{3y}{3} = \frac{x-6}{3}$

$f^{-1}(x) = \frac{1}{3}x - 2$



Example 4: Retailing Application.

A clerk needs to price a digital camera returned by a customer. The customer paid a total of $103.14, which included a gift-wrapping charge of $3 and 8% sales tax. What price should the clerk mark the tag?

Step 1: Write an equation for the total cost that models price as a function of cost.

$C = 1.08(p + 3)$ OR $C = 1.08p + 3.24$

Step 2: Find the inverse function that models price as a function of cost.

Solve for p. $C = 1.08p + 3.24$

$1.08p = C - 3.24$

$p = \frac{C - 3.24}{1.08}$

Step 3: Evaluate the inverse function for c = $103.14.

$p = \frac{103.14 - 3.24}{1.08}$ $p = 92.5$ $92.50