Algebra 2 Notes Name: ____________

Section 7.1 – Exponential Functions, Growth, and Decay

Moore's law, a rule used in the computer industry, states that the number of transistors per integrated circuit (the processing power) doubles every year. Beginning in the early days of integrated circuits, the growth in capacity may be approximated by this table.

Transistors per Integrated Chip							
Year	1965	1966	1967	1968	1969	1970	1971
Transistors	60	120	240	480	960	1920	3840

×2 ×2 ×2 ×2 ×2 ×2

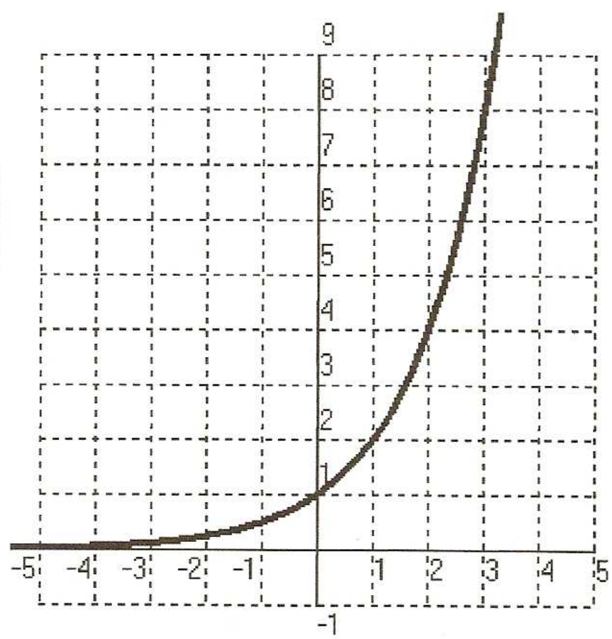
Growth that doubles every year can be modeled by using a function with a variable as an exponent. This function is known as an exponential function. The parent exponential function is $f(x) = b^x$, where the base b is a constant and the exponent x is the independent variable.

exponent

$$f(x) = b^x, \text{ where } b > 0,\ b \neq 1$$

base

The graph of the parent function $f(x) = 2^x$ is shown. The domain is $\mathbb{R}$ and the range is $y > 0$.



x	-2	-1	0	1	2	3
$f(x) = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Notice that as the x-values decrease, the graph of the function gets closer and closer to the x-axis. The function never reaches the x-axis because the value of 2^x cannot be zero. In this case, the x-axis is an asymptote. An asymptote is a line that a graphed function approaches as the value of x gets very large or very small.

A function of the form $y = ab^x$, with $a > 0$ and $b > 1$, is an exponential growth function, which increases as x increases. When $0 < b < 1$, the function is called an exponential decay function, which decreases as x increases.

growth 

decay 

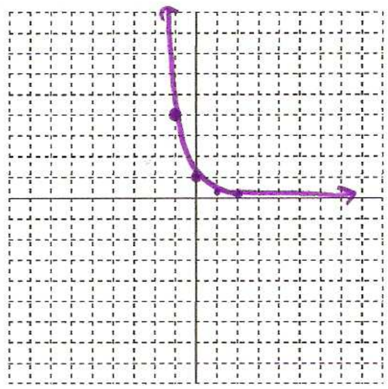
Example 1: Tell whether the function shows growth or decay. Then graph.

a. $f(x) = 0.25^x$ $\quad f(x) = \left(\frac{1}{4}\right)^x$

b

decay because $0 < b < 1$

x	y
-2	16
-1	4
0	1
1	$\frac{1}{4}$
2	$\frac{1}{16}$



D: $\mathbb{R}$

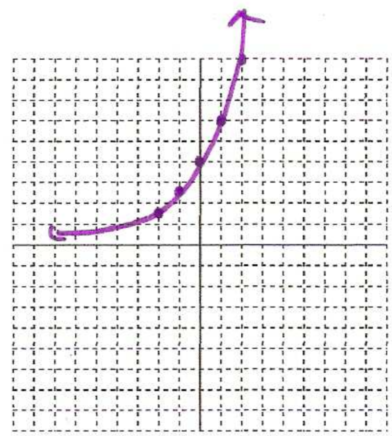
R: $y > 0$

b. $y = 4\left(\frac{3}{2}\right)^x$

b

growth because $b > 1$

x	y
-2	$\frac{16}{9} = 1\frac{7}{9}$
-1	$\frac{8}{3} = 2\frac{2}{3}$
0	4
1	6
2	9



D: $\mathbb{R}$

R: $y > 0$

You can model a growth or decay by a constant percent increase or decrease with the following formula:

$$A(t) = a(1 \pm r)^t$$

final amount

initial amount

rate of increase

* (change % to decimal!)

number of time periods

Example 2: Economics Applications.

a. Tony purchased a rare 1959 Gibson Les Paul guitar in 2000 for $12,000. Experts estimate that its value will increase by 14% per year. How much will the guitar be worth in 2004? When will the value of the guitar be $60,000? $t = \#$ years since 2000

$f(t) = 12000(1 + .14)^t$

$f(t) = 12000(1.14)^t$

In 2004:

$x = 4$

$f(4) = 12000(1.14)^4$

$f(4) \approx \boxed{\$20,267.52}$

Be \$60,000:

$\frac{60000}{12000} = \frac{12000(1.14)^t}{12000}$

$\underbrace{5}_{y_1} = \underbrace{1.14^t}_{y_2}$

calc intersection on calculator

$t \approx 12.3$ years so $\boxed{\approx 2012}$

b. The value of a truck bought new for $28,000 decreases by 9.5% each year. What will the truck be worth in 5 years? When will the truck value fall to $5000?

$f(t) = 28000(1 - 0.095)^t$

$f(t) = 28000(0.905)^t$

In 5 years:

$f(5) = 28000(0.905)^5$

$\approx \boxed{\$8216.12}$

Worth \$5000

$5000 = 28000(0.905)^t$

$\frac{5}{28} = (0.905)^t$

$t \approx \boxed{17.3 \text{ years}}$