Algebra 2 Notes Name: Lecy Section 8.6 - Radical Expressions & Radical Exponents

You are probably familiar with finding the square and square root of a number. These two operations are inverses of each other. Similarly, there are roots that correspond to larger powers.

- 5 and -5 are Square roots of 25 because $5^2 = 25$ and $(-5)^2 = 25$.
- 2 is the _____ root of 8 because $2^3 = 8$.
- 2 and -2 are fourth roots of 16 because $2^4 = 16$ and $\left(-2\right)^4 = 16$.
- a is the _____ roots of b if $a^n = b$.

The n th root of a real number a can be written as the radical expression $\frac{n}{2}$, where n is the $\frac{ndex}{n}$ of the radical and a is the $\frac{ndex}{n}$. When a number has more than one real root, the radical sign indicates only the $\frac{ndex}{n}$, or positive, root.

Numbers and Types of Real Roots			
Case	Roots	Example	
Odd index	1 real root	The real 3 rd root of 8 is 2.	
Even index; positive radicand	2 real roots	The real 4 th roots of 16 are \pm 2.	
Even index; negative radicand	O real roots	-16 has no real 4 th root.	
Radicand of 0	1 root of 0	The 3 rd root of 0 is 0.	

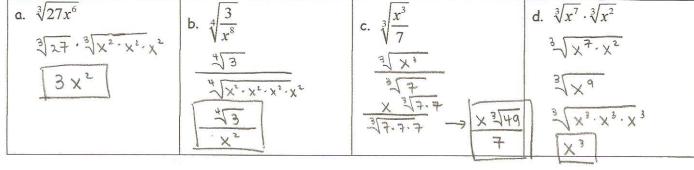
Example 1: Find all real roots.

a. fourth roots of 81	b. cube roots of -125	c. square root of -25
-3 and 3	- 5	neg # has no REAL Square root

The properties of square roots also apply to n th roots.

Properties of <i>n</i> th Roots				
For $a > 0$ and $b > 0$,				
Words	Numbers	Algebra		
Product Property of Roots The n th root of a product is equal to the product of the nn roots.	$\sqrt[3]{16} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$		
Quotient Property of Roots The nth root of a quotient is equal to the quotient of the nth root.	$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$		

Example 2: Simplify each expression. Assume that all variables are positive.



A	radical	exponent	is an exponent	that can be expresse	d as	- m		. where
m	and n are integers and n	(20)	Radical	expressions		written by	using	rational
ex	ponents.							

Rational Exponents				
For any natural number n and integer m ,				
Words	Numbers	Algebra		
The exponent $\frac{1}{n}$ indicates the n th root.	$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$	$a^{\frac{1}{n}} = \sqrt[n]{a}$		
The exponent $\frac{m}{n}$ indicated the n th root raised to the m th power.	$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$	$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$		

Example 3: Write the expression in radical form and then simplify.

2	1	5	1
a. $(-125)^{\frac{2}{3}}$	b. $64^{\frac{1}{3}}$	c. $4^{\frac{3}{2}}$	d. $32^{\frac{1}{5}}$
(3(-125)2	3/64	(2/4) ⁵	5/32
(-5)2	[4]	25	[2]

Example 4: Rewrite each expression using rational exponents. Then simplify when possible.

a. $\sqrt[4]{7^3}$	b. $\sqrt[3]{11^6}$	c. $\sqrt[4]{25^2}$
7 34	1/ 3	25
[≈ 4.30+]	11 2	25 2
	[121]	\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\

Rational exponents have the same properties as integer exponents. Look back in Section 1.5 in your book if you need to remember...

Example 5: Simplify each expression.

a. $25^{\frac{3}{5}} \cdot 25^{\frac{2}{5}}$	b. $\frac{8^{\frac{1}{3}}}{3}$	c. $36^{\frac{3}{8}} \cdot 36^{\frac{1}{8}}$
25	8 = 3	36 3 + 8
25 5		36 8
25	83	36 =
25	2	6