Algebra 2 Notes Name: Key
Section 8.6 - Radical Expressions & Radical Exponents

You are probably familiar with finding the square and square root of a number. These two operations are inverses of each other. Similarly, there are roots that correspond to larger powers.

5 and -5 are ___square___ roots of 25 because $5^2 = 25$ and $(-5)^2 = 25$.

2 is the ___cube___ root of 8 because $2^3 = 8$.

2 and -2 are ___fourth___ roots of 16 because $2^4 = 16$ and $(-2)^4 = 16$.

a is the ___n^{th}___ roots of b if $a^n = b$.

The n th root of a real number a can be written as the radical expression ___$\sqrt[n]{a}$___, where n is the ___index___ of the radical and a is the ___radicand___. When a number has more than one real root, the radical sign indicates only the ___principal___, or positive, root.

Numbers and Types of Real Roots		
Case	Roots	Example
Odd index	1 real root	The real 3rd root of 8 is 2.
Even index; positive radicand	2 real roots	The real 4th roots of 16 are ± 2.
Even index; negative radicand	0 real roots	-16 has no real 4th root.
Radicand of 0	1 root of 0	The 3rd root of 0 is 0.

Example 1: Find all real roots.

a. fourth roots of 81	b. cube roots of -125	c. square root of -25
-3 and 3	-5	neg # has no REAL square root

The properties of square roots also apply to n th roots.

Properties of n th Roots		
For $a > 0$ and $b > 0$,		
Words	Numbers	Algebra
Product Property of Roots The n th root of a product is equal to the product of the n n roots.	$\sqrt[3]{16} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
Quotient Property of Roots The n th root of a quotient is equal to the quotient of the n th root.	$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Example 2: Simplify each expression. Assume that all variables are positive.

a. $\sqrt[3]{27x^6}$	b. $\sqrt[4]{\frac{3}{x^8}}$	c. $\sqrt[3]{\frac{x^3}{7}}$	d. $\sqrt[3]{x^7} \cdot \sqrt[3]{x^2}$
$\sqrt[3]{27} \cdot \sqrt[3]{x^2 \cdot x^2 \cdot x^2}$ $\boxed{3x^2}$	$\frac{\sqrt[4]{3}}{\sqrt[4]{x^2 \cdot x^2 \cdot x^2 \cdot x^2}}$ $\boxed{\frac{\sqrt[4]{3}}{x^2}}$	$\frac{\sqrt[3]{x^3}}{\sqrt[3]{7}}$ $\frac{x\sqrt[3]{7 \cdot 7}}{\sqrt[3]{7 \cdot 7 \cdot 7}} \rightarrow \boxed{\frac{x\sqrt[3]{49}}{7}}$	$\sqrt[3]{x^7 \cdot x^2}$ $\sqrt[3]{x^9}$ $\sqrt[3]{x^3 \cdot x^3 \cdot x^3}$ $\boxed{x^3}$

A radical exponent is an exponent that can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$. Radical expressions can be written by using rational exponents.

Rational Exponents		
For any natural number n and integer m,		
Words	Numbers	Algebra
The exponent $\frac{1}{n}$ indicates the nth root.	$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$	$a^{\frac{1}{n}} = \sqrt[n]{a}$
The exponent $\frac{m}{n}$ indicated the nth root raised to the mth power.	$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$	$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Example 3: Write the expression in radical form and then simplify.

a. $(-125)^{\frac{2}{3}}$ $\left(\sqrt[3]{-125}\right)^2$ $(-5)^2$ $\boxed{25}$	b. $64^{\frac{1}{3}}$ $\sqrt[3]{64}$ $\boxed{4}$	c. $4^{\frac{5}{2}}$ $\left(\sqrt[2]{4}\right)^5$ 2^5 $\boxed{32}$	d. $32^{\frac{1}{5}}$ $\sqrt[5]{32}$ $\boxed{2}$

Example 4: Rewrite each expression using rational exponents. Then simplify ~~when possible~~.

a. $\sqrt[4]{7^3}$ $7^{\frac{3}{4}}$ $\boxed{\approx 4.304}$	b. $\sqrt[3]{11^6}$ $11^{\frac{6}{3}}$ 11^2 $\boxed{121}$	c. $\sqrt[4]{25^2}$ $25^{\frac{2}{4}}$ $25^{\frac{1}{2}}$ $\sqrt{25}$ $\boxed{5}$

Rational exponents have the same properties as integer exponents. Look back in Section 1.5 in your book if you need to remember...

Example 5: Simplify each expression.

a. $25^{\frac{3}{5}} \cdot 25^{\frac{2}{5}}$ $25^{\frac{3}{5}+\frac{2}{5}}$ $25^{\frac{5}{5}}$ 25^1 $\boxed{25}$	b. $\dfrac{8^{\frac{1}{3}}}{8^{\frac{2}{3}}}$ $8^{\frac{1}{3}-\frac{2}{3}}$ $8^{-\frac{1}{3}}$ $\dfrac{1}{8^{\frac{1}{3}}}$ $\boxed{\frac{1}{2}}$	c. $36^{\frac{3}{8}} \cdot 36^{\frac{1}{8}}$ $36^{\frac{3}{8}+\frac{1}{8}}$ $36^{\frac{4}{8}}$ $36^{\frac{1}{2}}$ $\boxed{6}$