

Algebra 2 Notes

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Section 6.4 - Factoring Polynomials

Recall that if a number is divided by any of its factors, the remainder is 0. Likewise, if a polynomial is divided by any of its factors, the remainder is zero.

The Remainder Theorem states that if a polynomial is divided by $(x-a)$, the remainder is the value of the function at a . So, if $(x-a)$ is a factor of $P(x)$, then $P(a) = 0$.

Factor Theorem	
Theorem	Example
For any polynomial $P(x)$, $(x-a)$ is a factor of $P(x)$ if and only if $P(a) = 0$.	Because $P(1) = 1^2 - 1 = 0$, $(x-1)$ is a factor of $P(x) = x^2 - 1$.

Example 1: Determine whether the given binomial is a factor of the polynomial $P(x)$.

a. $(x-3); P(x) = x^2 + 2x - 3$

$$\begin{array}{r|rrrr} 3 & 1 & 2 & -3 & \\ & \downarrow & 3 & 15 & \\ & 1 & 5 & 12 & \end{array}$$

$P(3) \neq 0$ so $(x-3)$ is NOT a factor of $P(x)$

* b. $(x+4); P(x) = 2x^4 + 8x^3 + 2x + 8$

$$\begin{array}{r|rrrrrr} -4 & 2 & 8 & 0 & 2 & 8 & \\ & \downarrow & -8 & 0 & 0 & -8 & \\ & 2 & 0 & 0 & 2 & 0 & \end{array}$$

$P(-4) = 0$ so $(x+4)$ is a factor of $P(x)$

You are already familiar with methods for factoring quadratic expressions. You can factor polynomials of higher degrees by using many of the methods you learned in Section 5.3.

Example 2: Factor by grouping.

a. $(x^3 + 3x^2)(-4x - 12)$

$$x^2(x+3) - 4(x+3)$$

$$(x+3)(x^2 - 4)$$

$$(x+3)(x+2)(x-2)$$

b. $(x^3 - 2x^2)(-9x + 18)$

$$x^2(x-2) - 9(x-2)$$

$$(x-2)(x^2 - 9)$$

$$(x-2)(x+3)(x-3)$$

c. $(2x^3 - x^2) + (8x - 4)$

$$x^2(2x-1) + 4(2x-1)$$

$$(2x-1)(x^2+4)$$

d. $x^4 - x^3 - 25x^2 + 25x$

$$x((x^3 - x^2) - 25x + 25)$$

$$x(x^2(x-1) - 25(x-1))$$

$$x(x-1)(x^2 - 25)$$

$$x(x-1)(x+5)(x-5)$$

Just as there is a special rule for factoring the difference of two squares, there are special rules for factoring the sum or difference of two cubes.

Factoring the Sum and the Difference of Two Cubes	
Method	Algebra
Sum of two cubes	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
Difference of two cubes	$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Example 2: Factor each expression.

a. $5x^4 + 40x$

$$5x(x^3 + 8)$$

$$5x((x)^3 + (2)^3)$$

$$5x(x+2)(x^2 - 2x + 4)$$

b. $8y^3 - 27$

$$(2y)^3 - (3)^3$$

$$(2y-3)(4y^2 + 6y + 9)$$

c. $x^6 - 64$

$$(x^2)^3 - (4)^3$$

$$(x^2 - 4)(x^4 + 4x^2 + 16)$$

$$(x-2)(x+2)(x^4 + 4x^2 + 16)$$

d. $1 + x^3$

$$(1)^3 + (x)^3$$

$$(1+x)(1-x+x^2)$$