Algebra 2 Notes

Name: Key

Section 5.2 – Dividing Polynomials

DAY ONE: Dividing Polynomials Using Long Division

Polynomial long division is a method for dividing a polynomial by another polynomial of a lower degree. It is very similar to dividing numbers.

Arithmetic Long Division	Polynomial Long Division
$\begin{array}{r} 23 + \frac{1}{12} \\ 12\overline{)277} \\ -\underline{24} \\ 37 \\ -\underline{36} \\ 1 \end{array}$	$\begin{array}{r} 2x+3 + \frac{1}{x+2} \\ x+2\overline{)2x^2+7x+7} \\ + \underline{-2x^2 - 4x} \\ 3x+7 \\ + \underline{-3x - 6} \\ 1 \end{array}$

IMPORTANT:

(1) Before using long division to divide two polynomials, make sure each polynomial is in standard form.

(2) Use a place-holder for any missing degree terms. For example, if given x^2-4, write x^2+0x-4.

Examples: Divide by using long division.

a. $(4x^2+3x^3+10)\div(x-2)$	b. $(15x^2+8x-12)\div(3x+1)$	c. $(3x^4-x^3+5x-1)\div(x+2)$
$\begin{array}{r} 3x^2+10x+20 \\ x-2\overline{)3x^3+4x^2+0x+10} \\ +\underline{-3x^3+6x^2} \\ 10x^2+0x \\ +\underline{-10x^2+20x} \\ 20x+10 \\ +\underline{-20x+40} \\ 50 \end{array}$	$\begin{array}{r} 5x+1 \\ 3x+1\overline{)15x^2+8x-12} \\ +\underline{-15x^2-5x} \\ 3x-12 \\ +\underline{-3x-1} \\ -13 \end{array}$	$\begin{array}{r} 3x^3-7x^2+14x-23 \\ x+2\overline{)3x^4-x^3+0x^2+5x-1} \\ +\underline{-3x^4-6x^3} \\ -7x^3+0x^2 \\ +\underline{+7x^3+14x^2} \\ 14x^2+5x \\ +\underline{-14x^2-28x} \\ -23x-1 \\ +\underline{+23x+46} \\ 45 \end{array}$
$\boxed{3x^2+10x+20+\frac{50}{x-2}}$	$\boxed{5x+1-\frac{13}{3x+1}}$	$\boxed{3x^3-7x^2+14x-23+\frac{45}{x+2}}$

DAY TWO: Dividing Polynomials Using Synthetic Division

Synthetic division is a shorthand method of dividing a polynomial by a <u>linear</u> binomial by using only the <u>Coefficients</u>. For synthetic division to work, the polynomial must be written in <u>Standard</u> form, using <u>0</u> as a coefficient for any missing terms, and the divisor must be in the form <u>$(x-a)$</u>.

Synthetic Division Method	
Divide $(2x^2+7x+9)\div(x+2)$ by using synthetic division.	
Words	Numbers
Step 1: Write the coefficients of the dividend, 2, 7, and 9. In the upper left corner, write the value of a for the divisor $(x-a)$. So $a=-2$. Copy the first coefficient in the dividend below the horizontal bar.	$\begin{array}{r\|rr\|r} -2 & 2 & 7 & 9 \\ & & & \\ \hline & 2 & & \end{array}$
Step 2: Multiply the first coefficient by the divisor, and write the product under the next coefficient. Add the numbers in the new column.	$\begin{array}{r\|rr\|r} -2 & 2 & 7 & 9 \\ & & -4 & \\ \hline & 2 & 3 & \end{array}$
REPEAT Step 2 until additions have been completed in all columns.	$\begin{array}{r\|rr\|r} -2 & 2 & 7 & 9 \\ & & -4 & -6 \\ \hline & 2 & 3 & 3 \end{array}$
Step 3: The quotient is represented by the numbers below the horizontal line. The boxed-in number is the remainder. The others are the coefficients of the polynomial quotient, in order of decreasing degree.	$=2x+3+\dfrac{3}{x+2}$

Example 1: Divide by using synthetic division.

a. $(4x^2-12x+9)\div\left(x+\frac{1}{2}\right)$	b. $(x^4-2x^3+3x+1)\div(x-2)$
$\begin{array}{r\|rr\|r} -\frac{1}{2} & 4 & -12 & 9 \\ & & -2 & 7 \\ \hline & 4 & -14 & 16 \end{array}$ $\boxed{4x-14+\dfrac{16}{x+\frac{1}{2}}}$	$\begin{array}{r\|rrrr\|r} 2 & 1 & -2 & 0 & 3 & 1 \\ & & 2 & 0 & 0 & 6 \\ \hline & 1 & 0 & 0 & 3 & 7 \end{array}$ $\boxed{x^3+3+\dfrac{7}{x-2}}$

You can use synthetic division to evaluate polynomials. The process is called synthetic substitution. The process of synthetic substitution is exactly the same as the process of synthetic division, but the final answer is interpreted differently; as described by the Remainder Theorem. (See next page.)

The Remainder Theorem

Theorem	Example
If the polynomial $P(x)$ is divided by $x-a$, then the remainder r is $P(a)$.	Divide x^3-4x^2+5x+1 by $x-3$.

$$\begin{array}{r|rrr|r} 3 & 1 & -4 & 5 & 1 \\ & & 3 & -3 & 6 \\ \hline & 1 & -1 & 2 & 7 \end{array}$$

$P(3)=7$

Example 2: Use synthetic substitution to evaluate the polynomial for the given value.

a. $P(x)=x^3-4x^2+3x-5$ for $x=4$

$$\begin{array}{r|rrr|r} 4 & 1 & -4 & 3 & -5 \\ & \downarrow & 4 & 0 & 12 \\ \hline & 1 & 0 & 3 & 7 \end{array}$$

$\boxed{P(4)=7}$

b. $P(x)=4x^4+2x^3+3x+5$ for $x=-\dfrac{1}{2}$

$$\begin{array}{r|rrrr|r} -\frac{1}{2} & 4 & 2 & 0 & 3 & 5 \\ & \downarrow & -2 & 0 & 0 & -\frac{3}{2} \\ \hline & 4 & 0 & 0 & 3 & \frac{7}{2} \end{array}$$

$\boxed{P\left(-\frac{1}{2}\right)=\frac{7}{2}}$

Example 4: Physics Application

A Van de Graaff generator is a machine that produces very high voltages by using small, safe levels of electric current. One machine has a current that can be modeled by $I(t)=t+2$, where $t>0$ represents time in seconds. The power of the system can be modeled by $P(t)=0.5t^3+6t^2+10t$. Write an expression that represents the voltage of the system given that $V=\dfrac{P}{I}$.

$$\begin{array}{r|rrr|r} -2 & 0.5 & 6 & 10 & 0 \\ & \downarrow & -1 & -10 & 0 \\ \hline & 0.5 & 5 & 0 & 0 \end{array}$$

$\boxed{V(t)=0.5t^2+5t}$

Question: Can you use synthetic division to divide a polynomial by x^2+3? Explain.

No; the divisor must be a linear binomial in the form $x-a$; x^2+3 is a quadratic binomial

You would have to use long division.