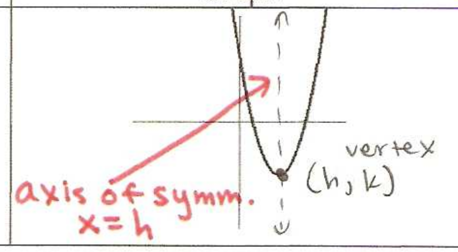
Algebra 2 Notes Name: Key

Section 5.2 – Properties of Quadratic Functions in Standard Form

When you transformed quadratic functions in the previous lesson, you saw that reflecting the parent function across the y-axis results in the same function. $f(x) = x^2$ and $g(x) = (-x)^2 = x^2$

This shows that parabolas are <u>Symmetric</u> curves. The <u>axis</u> of symmetry is the line through the vertex of the parabola that divides the parabola into two <u>congruent</u> halves.

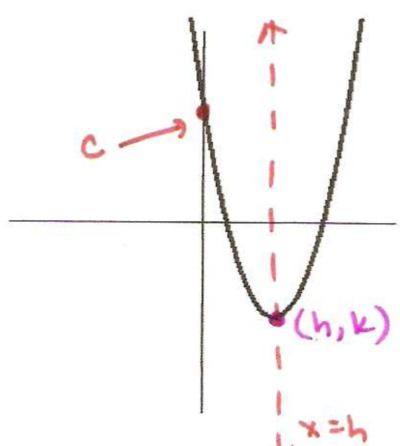
Axis of Symmetry – Quadratic Functions		
Words	**Algebra**	**Graph**
The axis of symmetry is a vertical line through the vertex of the function's graph.	The quadratic function $f(x) = a(x-h)^2 + k$ has the axis of symmetry $x = h$.	

Example 1: Identify the axis of symmetry for the following graphs.

a. $f(x) = 2(x+2)^2 - 3$ $f(x) = 2(x - -2)^2 - 3$ $h = -2$ $\boxed{x = -2}$	b. $y = (x-3)^2$ $h = 3$ $\boxed{x = 3}$	c. $f(x) = x^2 + 1$ $f(x) = (x-0)^2 + 1$ $h = 0$ $\boxed{x = 0}$

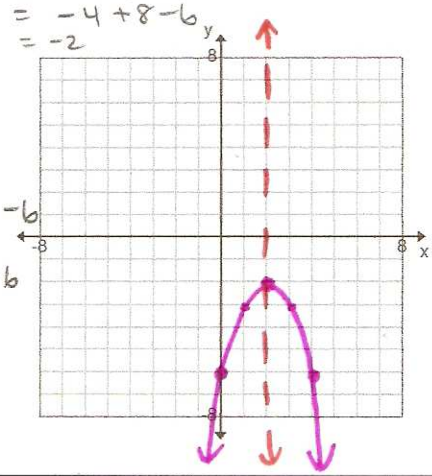
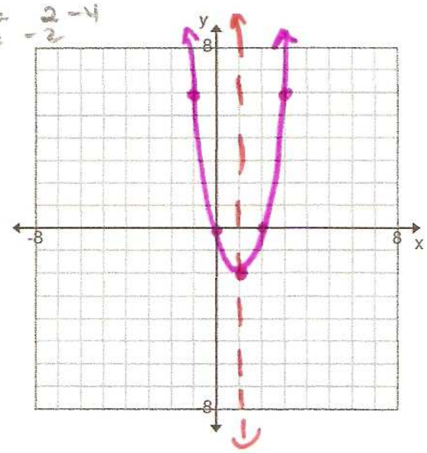
Another useful form of writing quadratic functions is the <u>Standard form</u>. The standard form of a quadratic function is $f(x) = ax^2 + bx + c$, where $a \neq 0$.

The coefficients of a, b, and c can show properties of the graph of the function. You can determine these properties by expanding the vertex form. SEE SUPPLEMENT PAGE, if interested.

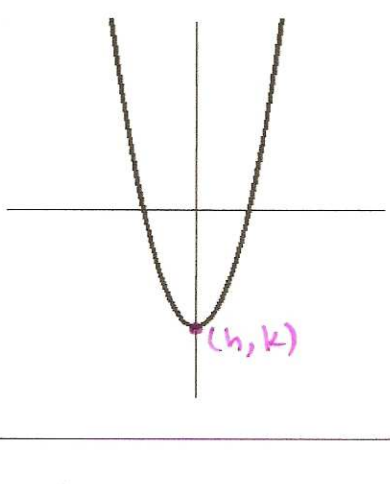
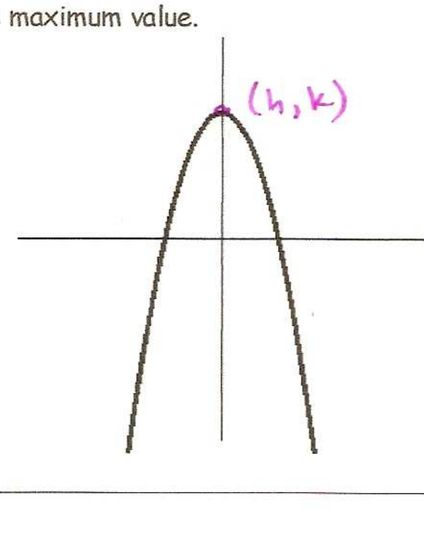
Properties of a Parabola
For $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$, the parabola has these properties: The parabola opens upward if <u>$a > 0$</u> and downward if <u>$a < 0$</u>. The axis of symmetry is the vertical line <u>$x = \frac{-b}{2a}$</u>. The vertex is the point <u>$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$</u> The y-intercept is <u>c</u>. 

$f(x) = ax^2 + bx + c$

Example 2: For each function, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function.

a. $f(x) = -x^2 + 4x - 6$ $\quad a=-1,\ b=+4,\ c=-6$	b. $f(x) = 2x^2 - 4x + 0$ $\quad a=2\ \ b=-4\ \ c=0$
ⓐ $a=-1$ negative opens downward ⓑ $x = \frac{-b}{2a}$ $x = \frac{-4}{2(-1)}$ $x = \frac{-4}{-2}$ $x = 2$ ⓒ $f(+2) = -(2)^2 + 4(2) - 6$ $= -4 + 8 - 6$ $= -2$ vertex $(2, -2)$ ⓓ y-int $= -6$ b/c $c = -6$ 	ⓐ $a=2$ positive opens up ⓑ $x = \frac{-b}{2a}$ $x = \frac{-(-4)}{2(2)}$ $x = \frac{4}{4}$ $x = 1$ ⓒ $f(1) = 2(1)^2 - 4(1)$ $= 2 - 4$ $= -2$ vertex $(1, -2)$ ⓓ y-int $= 0$ 

Substituting any real value of x into a quadratic equation results in a real number. Therefore, the domain of any quadratic function is all real numbers. The range of a quadratic function depends on its vertex and the direction the parabola opens.

Minimum and Maximum Values	
Opens Upward	Opens Downward
When a parabola opens upward, the y-value of the vertex is the minimum value. D: $\mathbb{R}$ R: $y \geq k$ 	When a parabola opens downward, the y-value of the vertex is the maximum value. D: $\mathbb{R}$ R: $y \leq k$ 

Example 3: Find the minimum or maximum value of each function. Then state its domain and range.

a. $f(x) = 2x^2 - 2x + 5$ $a = 2 \quad b = -2 \quad c = 5$

$x = \frac{--2}{2(2)}$ opens up

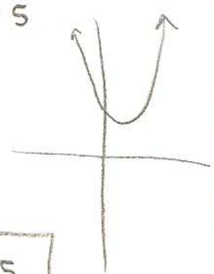
$x = \frac{2}{4}$ vertex $\left(\frac{1}{2}, 4\frac{1}{2}\right)$

$x = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 5$

$= \frac{1}{2} - 1 + 5$

$= 4.5$



$\boxed{\text{minimum value of } 4.5 \quad D: \mathbb{R} \quad R: y \geq 4.5}$

b. $f(x) = -2x^2 - 4$

$f(x) = -2x^2 + 0x - 4$ $a = -2 \quad b = 0 \quad c = -4$

$x = \frac{-b}{2a}$ opens down

$x = \frac{0}{-4}$ vertex $(0, -4)$

$x = 0$

$f(0) = -2(0)^2 - 4$

$f(0) = -4$

$\boxed{\text{max value of } -4 \quad D: \mathbb{R} \quad R: y \leq -4}$

Did you know your calculator will find minimum and maximum values of graphs for you? ☺

Check out the menu located by pressing the buttons 

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Example 4: Transportation Application

The power p in horsepower (hp) generated by a high-performance speedboat engine operating at r revolutions per minute (rpm) can be modeled by the function $p(r) = -0.0000147r^2 + 0.18r - 251$. What is the maximum power of this engine to the nearest horsepower? At how man y revolutions per minute must the engine be operating to achieve this power?

vertex @ $(6122.4, 300.0)$ used calculator

rpm   hp

$\boxed{\text{max power of } 300 \text{ hp @ } 6122 \text{ rpm}}$