Algebra 2 Notes

Name: Key

Section 5.1 – Using Transformations to Graph Quadratic Functions

A quadratic function is a function that can be written in the form $f(x) = a(x-h)^2 + k$ where $a \neq 0$. In a quadratic function, the variable is always squared.

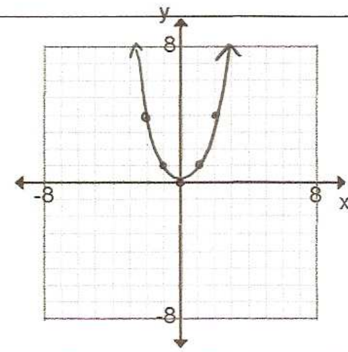
The Quadratic Parent Function $f(x) = x^2$

Domain: $\mathbb{R}$

Range: $y \geq 0$

Vertex: $(0, 0)$

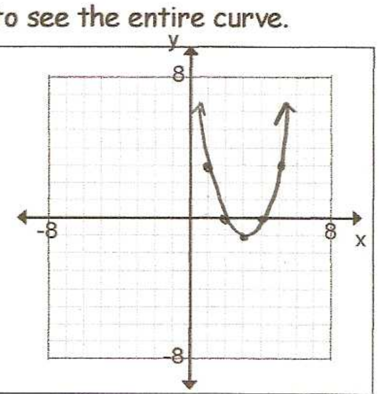
x	$y = x^2$
-2	4
-1	1
0	0
1	1
2	4



Notice that the graph of the parent function $f(x) = x^2$ is a U-shaped curve called a parabola. As with other functions, you can graph a quadratic function by plotting points with coordinates that make the equation true.

Example 1: Graph $f(x) = x^2 - 6x + 8$ by using a table. Plot enough points to see the entire curve.

x	$f(x) = x^2 - 6x + 8$	$(x, f(x))$
1	$f(1) = 1^2 - 6(1) + 8 = 3$	$(1, 3)$
2	$f(2) = 2^2 - 6(2) + 8 = 0$	$(2, 0)$
3	$f(3) = 3^2 - 6(3) + 8 = -1$	$(3, -1)$
4	$f(4) = 16 - 24 + 8 = 0$	$(4, 0)$
5	$f(5) = 25 - 30 + 8 = 3$	$(5, 3)$

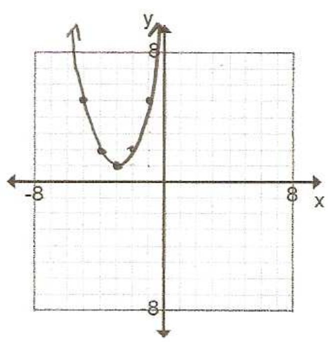


You can graph quadratic functions by applying transformations to the parent function $f(x) = x^2$.

Example 2: Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph.

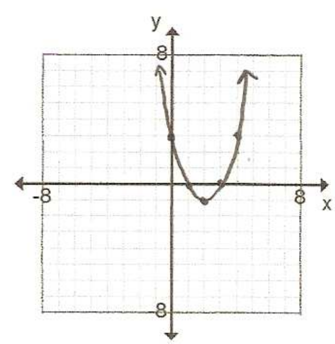
a. $g(x) = (x+3)^2 + 1$

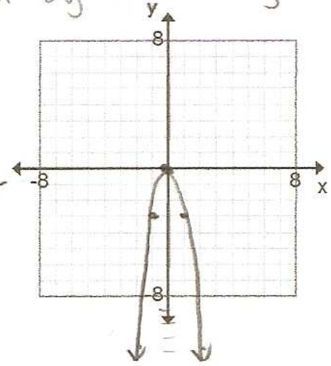
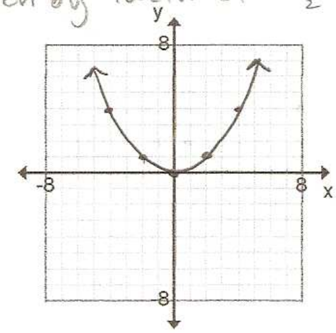
3 units left
1 unit up



b. $g(x) = (x-2)^2 - 1$

2 units right
1 unit down



c. $g(x) = -3x^2$ reflected across x-axis vertical stretch by factor of 3 over 1 down 3 over 2 down 12 	d. $g(x) = \left(\frac{1}{2}x\right)^2$ horizontal stretch by factor of $\frac{1}{\frac{1}{2}} = 2$ over 2 up 1 over 4 up 4 

If a parabola opens upward, it has a <u>minimum</u> point. If a parabola opens downward, it has a <u>maximum</u> point. This lowest or highest point is the <u>vertex</u> of a parabola.

The parent function $f(x) = x^2$ has its vertex at the <u>origin</u>. You can identify the vertex of the other quadratic functions by analyzing the function in <u>vertex</u> form. The vertex form of a quadratic function is $f(x) = a(x-h)^2 + k$, where a, h, and k are constants.

$$f(x) = a(x-h)^2 + k$$

indicates reflection across x-axis and/or a vertical stretch or compression

h indicates horiz. translation ⟷

k indicates vert. translation ↕

Because the vertex is translated h horizontal units and k vertical units from the <u>origin</u>, the vertex of the parabola is at <u>(h, k)</u>.

Example 3: Use the description to write the quadratic function in vertex form. Check w/ calculator. ☺

a.	b.	c.
a. The parent function $f(x) = x^2$ is reflected across the x-axis, vertically stretched by a factor of 6, and translated 3 units right to create g. vertex at (3, 0) $\boxed{g(x) = -6(x-3)^2}$	b. The parent function $f(x) = x^2$ is vertically compressed by a factor of $\frac{1}{3}$ and translated 2 units right and 4 units down to create g. vertex (2, -4) $\boxed{g(x) = \frac{1}{3}(x-2)^2 - 4}$	c. The parent function $f(x) = x^2$ is reflected across the x-axis and translated 5 units left and 1 unit up to create g. vertex (-5, 1) $f(x) = -(x - -5)^2 + 1$ $\boxed{f(x) = -(x+5)^2 + 1}$