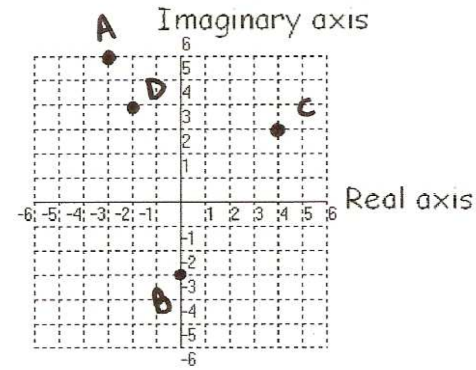
Algebra 2 Notes — Name: Key

Section 4.4 - Operations with Complex Numbers

Just as you can represent real numbers graphically as points on a number line, you can represent complex numbers in a special coordinate plane. The Complex plane is a set of coordinate axes in which the horizontal axis represents real numbers and the vertical axis represents imaginary numbers.

Example 1: Graph each complex number.

	real	imaginary
A. $-3+6i$	-3	6
B. $-3i$ $0-3i$	0	-3
C. $4+3i$	4	3
D. $-2+4i$	-2	4



Absolute Value of a Complex Number		
Words	**Algebra**	**Example**
The absolute value of a complex number $a+bi$ is the distance from the origin to the point (a,b) in the complex plane, and is denoted $\lvert a+bi\rvert$.	$\lvert a+bi\rvert=\sqrt{a^2+b^2}$	$\lvert 3+4i\rvert=\sqrt{3^2+4^2}$ $=\sqrt{9+16}$ $=\sqrt{25}$ $=5$

Example 2: Find the absolute value of each complex number. Show your work!

a. $\lvert -9+3i\rvert$	b. $\lvert 6\rvert = \lvert 6+0i\rvert$	c. $\lvert -5i\rvert = \lvert 0-5i\rvert$
$=\sqrt{(-9)^2+3^2}$	$=\sqrt{6^2+0^2}$	$=\sqrt{0^2+(-5)^2}$
$=\sqrt{81+9}$	$=\sqrt{36+0}$	$=\sqrt{0+25}$
$=\sqrt{90}$	$=\sqrt{36}$	$=\sqrt{25}$
$=\sqrt{9\cdot 10} = \boxed{3\sqrt{10}}$	$=\boxed{6}$	$=\boxed{5}$

Adding and subtracting complex numbers is similar to adding and subtracting variable expressions with like terms. Simply combine the real parts, and combine the imaginary parts. The set of complex numbers has all the properties of the set of real numbers. So you can use the Commutative, Associative, and Distributive Properties to simplify complex expressions.

Example 3: Add or subtract. Write the result in the form $a+bi$.

a. $(-2+4i)+(3-11i)$	b. $(4-i)+(5+8i)$	c. $(10+3i)+(10-4i)$
$(-2+3)+(4i+-11i)$	$(4+-5)+(-i+-8i)$	$(10+-10)+(3i+4i)$
$\boxed{1-7i}$	$\boxed{-1-9i}$	$0+7i$
		$\boxed{7i}$

You can multiply complex numbers by using the Distributive Property and treating the imaginary parts as like terms. Simplify further by using the fact that $i^2 = -1$.

Example 4: Multiply. Write the result in the form $a+bi$.

a. $2i(3-5i)$ $6i - 10i^2$ $6i - 10(-1)$ $6i + 10$ $\boxed{10 + 6i}$	b. $(5-6i)(4+3i)$ $20 + 15i - 24i - 18i^2$ $20 - 9i - 18(-1)$ $20 - 9i + 18$ $\boxed{38 - 9i}$	c. $(3+2i)^2 = (3+2i)(3+2i)$ $9 + 6i + 6i + 4i^2$ $9 + 12i + 4(-1)$ $9 + 12i - 4$ $\boxed{5 - 12i}$

The imaginary unit i can be raised to higher powers as shown below.

Powers of i		
$i^1 = 1$	$i^5 = i^4 \cdot i = \underline{1} \cdot \underline{i} = \underline{i}$	$i^9 = (i^2)^4 \cdot i = i$
$i^2 = -1$	$i^6 = i^4 \cdot i^2 = \underline{1} \cdot \underline{-1} = \underline{-1}$	$i^{10} = (i^2)^5 = (-1)^5 = -1$
$i^3 = i^2 \cdot i = -1 \cdot i = -i$	$i^7 = i^4 \cdot i^3 = \underline{1} \cdot \underline{-i} = \underline{-i}$	$i^{11} = (i^2)^5 \cdot i = -i$
$i^4 = i^2 \cdot i^2 = -1 \cdot (-1) = 1$	$i^8 = i^4 \cdot i^4 = \underline{1} \cdot \underline{1} = \underline{1}$	$i^{12} = (i^2)^6 = 1$

Example 5: Simplify.

a. $-3i^{16}$ $-3(i^2)^8$ $-3(-1)^8$ $-3 \cdot 1$ $\boxed{-3}$	b. i^{25} $i^{24} \cdot i$ $(i^2)^{12} \cdot i$ $(-1)^{12} \cdot i$ $1 \cdot i$ $\boxed{i}$	c. $-\frac{1}{2}i^{43}$ $-\frac{1}{2} \cdot i^{42} \cdot i$ $-\frac{1}{2}(i^2)^{21} \cdot i$ $-\frac{1}{2}(-1)^{21} \cdot i = -\frac{1}{2} \cdot -1 \cdot i = \boxed{\frac{i}{2}}$

Recall that expressions in simplest form cannot have square roots in the denominator. Because the imaginary unit represents a square root, you must <u>rationalize</u> any denominator that contains an imaginary unit. To do this, multiply the numerator and denominator by the complex conjugate of the denominator.

What is the complex conjugate? Well, if the complex number is $a+bi$, then its conjugate is <u>$a - bi$</u>.

Example 6: Simplify.

a. $\frac{6}{i} \cdot \frac{i}{i}$ $\frac{6i}{i^2}$ $\frac{6i}{-1}$ $\boxed{-6i}$	b. $\frac{3+7i}{8i} \cdot \frac{i}{i}$ $\frac{3i + 7i^2}{8i^2}$ $\frac{3i + 7(-1)}{8(-1)}$ $\frac{3i - 7}{-8}$ $\boxed{\frac{7}{8} - \frac{3}{8}i}$ OR $\frac{7 - 3i}{8}$	c. $\frac{1}{-2-3i} \cdot \frac{-2+3i}{-2+3i}$ $\frac{-2+3i}{4 - 6i + 6i - 9i^2}$ $\frac{-2+3i}{4 - 9(-1)}$ $\frac{-2+3i}{4+9}$ $\frac{-2+3i}{13}$ OR $\boxed{\frac{-2}{13} + \frac{3}{13}i}$	d. $\frac{5-i}{2+4i} \cdot \frac{2-4i}{2-4i}$ $\frac{10 - 20i - 2i + 4i^2}{4 - 8i + 8i - 16i^2}$ $\frac{10 - 22i - 4}{4 + 16}$ $\frac{6 - 22i}{20}$ $\frac{3 - 11i}{10}$ OR $\boxed{\frac{3}{10} - \frac{11i}{10}}$