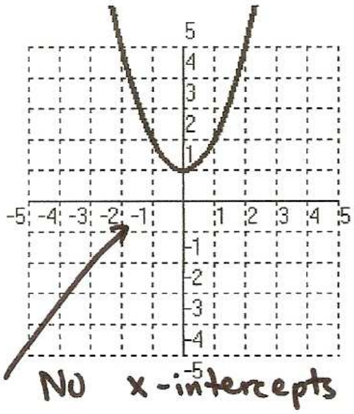
Algebra 2 Notes

Name: Key

Section 4.4 Complex Numbers and Roots



You can see in the graph of $f(x)=x^2+1$ that f has no real zeros. If you solve the corresponding equation $0=x^2+1$, you find that $x=\pm\sqrt{-1}$, which has no real solutions. However, you can find solutions if you define the square root of negative numbers, which is why imaginary numbers were invented. The imaginary unit i is defined as $\sqrt{-1}$. You can use the imaginary unit to write the square root of any negative number.

Imaginary Numbers		
Words	**Numbers**	**Algebra**
An imaginary number is the square root of a negative number.	$\sqrt{-1}=1$	If b is a positive real number, then $\sqrt{-b}=i\sqrt{b}$ and $\sqrt{-b^2}=bi$.
Imaginary numbers can be written in the form bi, where b is a real number and i is the imaginary unit.	$\sqrt{-2}=\sqrt{-1}\cdot\sqrt{2}=i\sqrt{2}$ $\sqrt{-4}=\sqrt{-1}\cdot\sqrt{4}=2i$	
The square of an imaginary number is the original negative number.	$\left(\sqrt{-1}\right)^2=i^2=-1$	$\left(\sqrt{-b}\right)^2=-b$

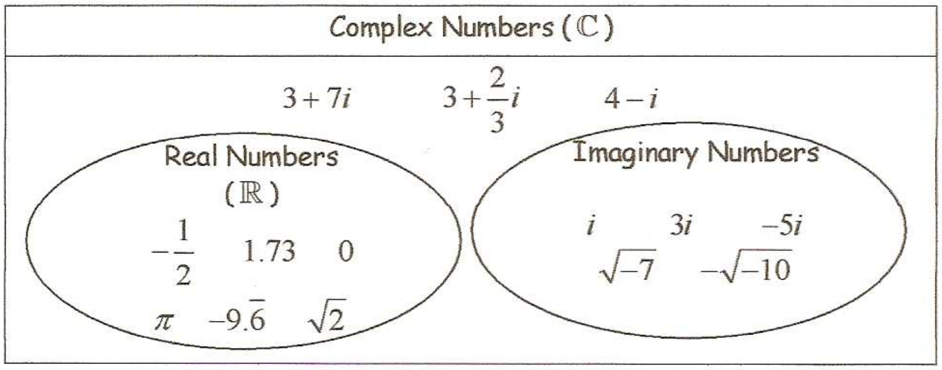
Example 1: Express each number in terms of i.

a. $3\sqrt{-16}$ $3\sqrt{-1}\cdot\sqrt{16}$ $3i\cdot 4$ $\boxed{12i}$	b. $-\sqrt{-75}$ $-\sqrt{-1}\cdot\sqrt{75}$ $-i\cdot\sqrt{25\cdot 3}$ $-i\cdot 5\sqrt{3}$ $\boxed{-5i\sqrt{3}}$	c. $\sqrt{-12}$ $\sqrt{-1}\cdot\sqrt{4\cdot 3}$ $i\cdot 2\sqrt{3}$ $\boxed{2i\sqrt{3}}$
d. $2\sqrt{-36}$ $2\sqrt{-1}\cdot\sqrt{36}$ $2i\cdot 6$ $\boxed{12i}$	e. $-\frac{1}{3}\sqrt{-63}$ $-\frac{1}{3}\sqrt{-1}\cdot\sqrt{9\cdot 7}$ $-\frac{1}{3}i\cdot 3\sqrt{7}$ $\boxed{-i\sqrt{7}}$	f. $-2\sqrt{-96}$ $-2\sqrt{-1}\cdot\sqrt{96}$ $-2i\sqrt{16\cdot 6}$ $-2i\cdot 4\sqrt{6}$ $\boxed{-8i\sqrt{6}}$

Example 2: Solve each equation.

a. $\sqrt{x^2}=\sqrt{-81}$ $x=\pm\sqrt{-1}\cdot\sqrt{81}$ $\boxed{x=\pm 9i}$	b. $3x^2+75=0$ $3x^2=-75$ $\sqrt{x^2}=\sqrt{-25}$ $\boxed{x=\pm 5i}$

c. $x^2 + 48 = 0$	d. $9x^2 - 10 = -35$
$\sqrt{x^2} = \sqrt{-48}$ $x = \pm i\sqrt{16 \cdot 3}$ $\boxed{x = \pm 4i\sqrt{3}}$	$9x^2 = -25$ $\sqrt{x^2} = \sqrt{\frac{-25}{9}}$ $\boxed{x = \pm \frac{5}{3}i \quad \text{OR} \quad x = \pm \frac{5i}{3}}$



A complex number is a number that can be written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. The set of real numbers is a subset of the set of complex numbers $\mathbb{C}$.

Every complex number has a real part a and an imaginary part b.

$$\underbrace{a}_{\text{real part}} + \underbrace{b}_{\text{imaginary part}} i$$

Real numbers are complex numbers where $b = 0$. Imaginary numbers are complex numbers where $a = 0$ and $b \neq 0$. These are sometimes called pure imaginary numbers.

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.

Example 3: Find the values of x and y that make each equation true.

a. $3x - 5i = 6 + (10y)i$	b. $2x - 6i = -8 + (20y)i$	c. $-8 + (6y)i = 5x - i\sqrt{6}$
$3x = 6$ $\boxed{x = 2}$ $-5 = -(10y)$ $5 = 10y$ $\boxed{y = \frac{1}{2}}$	$2x = -8$ $\boxed{x = -4}$ $-6 = 20y$ $\boxed{-\frac{3}{10} = y}$	$5x = -8$ $\boxed{x = -\frac{8}{5}}$ $6y = -\sqrt{6}$ $\boxed{y = -\frac{\sqrt{6}}{6}}$