Algebra 2 Notes Name: Key
Section 3.6 – Multiplying Matrices

DAY ONE:

In Section 4.1, you multiplied matrices by a number called a scalar. Let's review this idea now.

Example 1: *Simplify each matrix expression.*

a. $-4\begin{bmatrix} -2 & 3 \\ 0 & 1 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 8 & -12 \\ 0 & -4 \\ -4 & 12 \end{bmatrix}$	b. $\frac{1}{3}\begin{bmatrix} -1 & 3 & 6 \\ 0 & -4 & 9 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & 1 & 2 \\ 0 & -\frac{4}{3} & 3 \end{bmatrix}$

You can also multiply matrices together. The product of two or more matrices is the matrix product.
The following rules apply when multiplying matrices.

Rule 1:

Matrices A and B can be multiplied together as AB only if the number of columns in A equals the number of rows in B.

Need a helpful hint?

The CAR key: **C**olumns (of A) **A**s **R**ows (of B) or matrix product AB won't even start.

Rule 2:

The product of an $m \times n$ matrix and an $n \times p$ matrix is an $m \times p$ matrix.

Example 1:

Given matrices $P_{2\times5}$, $Q_{5\times3}$, $R_{4\times3}$, and $S_{4\times5}$, tell whether each product is defined. If so, give its dimensions.

a. PQ	b. RS	c. QP	d. SR	e. SQ
$2\times 5 \quad 5\times 3$	$4\times\cancel{3 \quad 4}\times 5$	$5\times\cancel{3 \quad 2}\times 5$	$4\times\cancel{5 \quad 4}\times 3$	$4\times 5 \quad 5\times 3$
$\boxed{\text{yes; } 2\times 3}$	$\boxed{\text{no}}$	$\boxed{\text{no}}$	$\boxed{\text{no}}$	$\boxed{\text{yes; } 4\times 3}$

Just as you look across the columns of A and down the rows of B to see if a product AB exists, you do the same to find the entries in a matrix product.

Multiplying Matrices

Words	Numbers	Algebra
In a matrix $P = AB$, each element p_{ij} is the sum of the products of consecutive entries in row i in matrix A and the column j in matrix B.	$P = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} =$ $\begin{bmatrix} 1\cdot5+2\cdot7 & 1\cdot6+2\cdot8 \\ 3\cdot5+4\cdot7 & 3\cdot6+4\cdot8 \end{bmatrix}$	$P = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}\begin{bmatrix} c_1 & c_2 \\ d_1 & d_2 \end{bmatrix} =$ $\begin{bmatrix} a_1\cdot c_1 + a_2\cdot d_1 & a_1\cdot c_2 + a_2\cdot d_2 \\ b_1\cdot c_1 + b_2\cdot d_1 & b_1\cdot c_2 + b_2\cdot d_2 \end{bmatrix}$

Seem like a confusing mess? Well, how you line up your matrices when multiplying them will help a great deal with your organization of your work.

Example 2: Multiply the following matrices, if possible.

a. $A=\begin{bmatrix}0 & 3\\ -1 & -4\\ 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}4 & 0\\ -2 & 1\end{bmatrix}$. Find AB.

$3\times 2 \quad 2\times 2 \quad = 3\times 2$

$$\begin{bmatrix}4 & 0\\ -2 & 1\end{bmatrix}$$

$$\begin{bmatrix}0 & 3\\ -1 & -4\\ 1 & 2\end{bmatrix}\begin{bmatrix}0\cdot 4+3\cdot -2 & 0\cdot 0+3\cdot 1\\ -1\cdot 4+-4\cdot -2 & -1\cdot 0+-4\cdot 1\\ 1\cdot 4+2\cdot -2 & 1\cdot 0+2\cdot 1\end{bmatrix}$$

$$\begin{bmatrix}0+-6 & 0+3\\ -4+8 & 0+-4\\ 4+-4 & 0+2\end{bmatrix}$$

$$\boxed{\begin{bmatrix}-6 & 3\\ 4 & -4\\ 0 & 2\end{bmatrix}}$$

b. $C=\begin{bmatrix}4 & -2\\ 5 & -4\end{bmatrix}$ and $D=\begin{bmatrix}8 & 0 & -1 & 0\\ 2 & -5 & 1 & 8\end{bmatrix}$. Find CD.

$2\times 2 \quad 2\times 4 \quad 2\times 4$

$$\begin{bmatrix}8 & 0 & -1 & 0\\ 2 & -5 & 1 & 8\end{bmatrix}$$

$$\begin{bmatrix}4 & -2\\ 5 & -4\end{bmatrix}\begin{bmatrix}32+-4 & 0+10 & -4+-2 & 0+-16\\ 40+-8 & 0+20 & -5+-4 & 0+-32\end{bmatrix}$$

$$\boxed{\begin{bmatrix}28 & 10 & -6 & -16\\ 32 & 20 & -9 & -32\end{bmatrix}}$$

c. $U=\begin{bmatrix}-6 & 7\\ 0 & 1\end{bmatrix}$ and $T=\begin{bmatrix}1 & 0 & -1\\ 5 & -7 & 3\end{bmatrix}$. Find TU.

$2\times 3 \quad 2\times 2$

$\boxed{\text{undefined}}$

d. $A=\begin{bmatrix}3 & -1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}4\\ -1\\ 1\end{bmatrix}$. Find AB.

$1\times 3 \quad 3\times 1 \quad = 1\times 1$

$$\begin{bmatrix}4\\ -1\\ 1\end{bmatrix}$$

$$\begin{bmatrix}3 & -1 & 0\end{bmatrix}\begin{bmatrix}12+1+0\end{bmatrix}$$

$$\boxed{\begin{bmatrix}13\end{bmatrix}}$$

DAY TWO:

Example 1: Use matrix multiplication and equal matrices to find x and y.

$$\begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} x & 3 \\ -3 & 2y \end{bmatrix}=\begin{bmatrix} 3 & -6 \\ -4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} x & 3 \\ -3 & 2y \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0+3 & 0+-2y \\ 2x+0 & 6+0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2y \\ 2x & 6 \end{bmatrix}$$

So $\rightarrow \begin{bmatrix} 3 & -2y \\ 2x & 6 \end{bmatrix}=\begin{bmatrix} 3 & -6 \\ -4 & 6 \end{bmatrix}$

$$\frac{-2y}{-2}=\frac{-6}{-2}$$

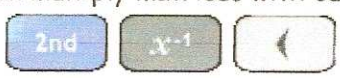
$$\boxed{y=3}$$

$$\frac{2x}{2}=\frac{-4}{2}$$

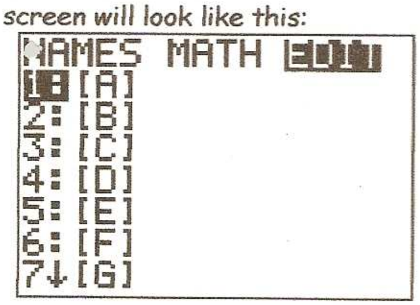
$$\boxed{x=-2}$$

We are now going to learn how to use our calculator to multiply matrices. There is a need to know how to multiply matrices by hand. For example, the previous example is one where you would have to multiply the matrices by hand in order to solve the problem, since some of the matrix entries are variables.

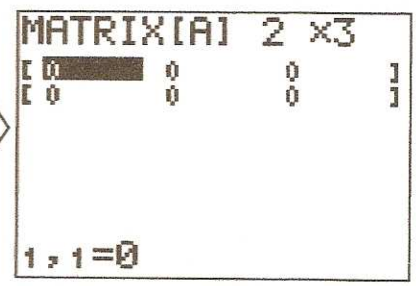
Before we can multiply matrices with our calculator, we need to enter the matrices. You do this by pressing the buttons



to pull up the screen where we can edit matrices. Your calculator screen will look like this:



You will need to select the matrix name you want to enter, and then select the appropriate dimensions for the matrix in the screen at right.



Once you have your matrices entered, you can go back to your home screen before typing in the matrix operations you wish to do. To call up a matrix you entered, press

and then select the desired matrix from the name menu. If you need more help, a tutorial video will be on the website.

Example 2: Use the matrices to perform the following matrix operations, if possible.

$$A=\begin{bmatrix} 7 & 3 \\ -2 & 0 \end{bmatrix} \quad B=\begin{bmatrix} 2 & 4 & 1 \\ 5 & 0 & -2 \\ 1 & -1 & 3 \end{bmatrix} \quad C=\begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & -2 \end{bmatrix} \quad D=\begin{bmatrix} -2 & 5 & 0 \end{bmatrix} \quad I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

a. AC	b. CB	d. $2BC$	e. $\frac{1}{2}IC$
$\begin{bmatrix} 13 & 0 & 1 \\ -2 & 0 & -2 \end{bmatrix}$	$\begin{bmatrix} 3 & 3 & 4 \\ 2 & 10 & -4 \end{bmatrix}$	Not possible	$\begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & -1 \end{bmatrix}$