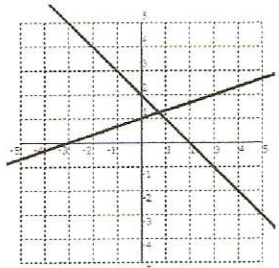
Algebra 2 Notes Name: Izzy

Section 3.2 – Using Algebraic Methods to Solve Linear Systems



The graph shows a system of linear equations. As you can see, without the use of technology, determining the solution from the graph is not easy. You can use the Substitution method to find an exact solution. In Substitution, you solve on equation for one variable and then substitute this expression into the other equation.

Example 1: Use substitution to solve each system of equations.

a. $\begin{cases} y = x+2 \\ x+y=8 \end{cases}$ $x + (x+2) = 8$ $2x+2=8$ $-2 \quad -2$ $2x=6$ $\frac{2x}{2} = \frac{6}{2}$ $x=3$ $y=3+2$ $y=5$ $\boxed{(3,5)}$	b. $\begin{cases} 2x+y=6 \\ y-8x=1 \end{cases}$ $+8x \quad +8x$ $y = 8x+1$ $2x + (8x+1) = 6$ $10x+1=6$ $-1 \quad -1$ $\frac{10x}{10} = \frac{5}{10}$ $x=\frac{1}{2}$ $2(\frac{1}{2}) + y = 6$ $1 + y = 6$ $-1 \quad -1$ $y=5$ $\boxed{(\frac{1}{2},5)}$	c. $\begin{cases} 5x+6y=-9 \\ 2x-2=-y \end{cases}$ $-1 \quad -1$ $y = -2x+2$ $5x+6(-2x+2) = -9$ $5x + -12x + 12 = -9$ $-12 \quad -1$ $-7x+12=-9$ $\frac{-7x}{-7} = \frac{-21}{-7}$ $x=3$ $\boxed{(3,-4)}$ $y=-2(3)+2$ $y=-6+2$ $y=-4$

You can also solve systems of equations with the elimination method. With elimination, you get rid of one of the variables by adding or subtracting equations. You may have to multiply one or both equations by a number to create variable terms that can be eliminated.

Example 2: Use elimination to solve each system of equations.

a. $\begin{cases} 2x+\cancel{3y}=34 \\ 4x-\cancel{3y}=-4 \end{cases}$ + $6x = 30$ $\frac{6x}{6} = \frac{30}{6}$ $x=5$ $2(5)+3y=34$ $10+3y=34$ $-10 \quad -10$ $\frac{3y}{3} = \frac{24}{3}$ $y=8$ $\boxed{(5,8)}$	b. $3(2x+4y=-10)$ $-2(3x+3y=-3)$ $\cancel{6x}+12y=-30$ $+ \cancel{-6x}-6y=6$ $6y=-24$ $y=-4$ $2x+4(-4)=-10$ $2x-16=-10$ $+16 \quad +16$ $\frac{2x}{2} = \frac{6}{2}$ $x=3$ $\boxed{(3,-4)}$	c. $2(x-2y=-6)$ $3x+4y=+2$ $2x-\cancel{4y}=-12$ $+ \ 3x+\cancel{4y}=+2$ $5x=-10$ $x=-2$ $-2-2y=-6$ $+2 \quad +2$ $\frac{-2y}{-2} = \frac{-4}{-2}$ $y=2$ $\boxed{(-2,2)}$

Systems may have infinitely many or no solutions. When you try to solve these systems algebraically, the result will be an identity or a contradiction.

A Consistent system is a set of equations or inequalities that has at least one solution, and an inconsistent system will have no solutions.

An independent system is one that has one solution. A dependent system is one that has infinitely many solutions.

Example 3: Classify the system and determine the number of solutions.

a. $-3\left(2x + y = 8\right)$
$\quad 6x + 3y = -15$

$$\begin{aligned} -\cancel{6x} - \cancel{3y} &= -24 \\ + \quad \cancel{6x} + \cancel{3y} &= -15 \\ \hline 0 &= -39 \end{aligned}$$

FALSE

$\boxed{\text{inconsistent; no sol'n}}$

b. $56x + 8y = -32$
$-8\left(7x + y = -4\right)$

$$\begin{aligned} \cancel{56x} + \cancel{8y} &= \cancel{-32} \\ + \quad -\cancel{56x} - \cancel{8y} &= \cancel{32} \\ \hline 0 &= 0 \end{aligned}$$

TRUE

$\boxed{\text{consistent, dependent; infinite sol'ns}}$

Example 4: Application

A zookeeper needs to mix feed for the prairie dogs so that the feed has the right amount of protein. Feed A has 12% protein. Feed B has 5% protein. How many pounds of each does he need to get 100 pounds of feed that is 8% protein?

x = # lbs Feed A

y = # lbs Feed B

$0.12x + 0.05y = 0.08(100)$

$-0.12(x + y = 100)$

$$\begin{aligned} \cancel{0.12x} + 0.05y &= 8 \\ + \quad \cancel{-0.12x} - 0.12y &= -12 \\ \hline -0.07y &= -4 \\ \overline{-0.07} \quad & \quad \overline{-0.07} \end{aligned}$$

$y \approx 57.1$

$x + 57.1 = 100$
$\quad -57.1 \quad -57.1$

$x \approx 42.9$

$\boxed{\approx 42.9 \text{ lbs Feed A and } 57.1 \text{ lbs Feed B}}$