Algebra 2 Notes

Section 10.3 - Ellipses

Name: key

DAY ONE:

An ellipse is the set of all points $P(x,y)$ in a plane such that the sum of the distances from any point P on the ellipse to two fixed points F_1 and F_2, called the foci, (singular: focus), is the constant sum $d = PF_1 + PF_2$. This distance d can be represented by the length of a piece of string connecting two pushpins located at the foci.

You can use the distance formula to find the constant sum of an ellipse.

Example 1: Use the Distance Formula to find the constant sum of an ellipse with...

foci $F_1(-3,0)$ and $F_2(3,0)$, and the point on the ellipse $(0,4)$

$d = PF_1 + PF_2$

$d = \sqrt{(0--3)^2+(4-0)^2} + \sqrt{(0-3)^2+(4-0)^2}$

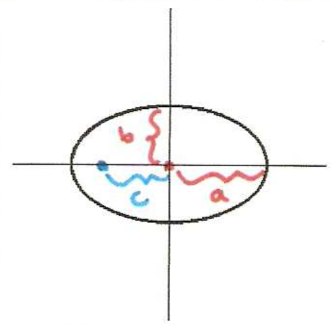
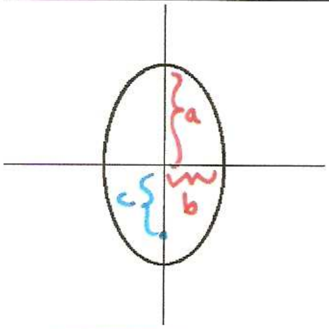
$d = \sqrt{9+16} + \sqrt{9+16}$

$d = \sqrt{25} + \sqrt{25}$

$d = 5 + 5$

$\boxed{d = 10}$

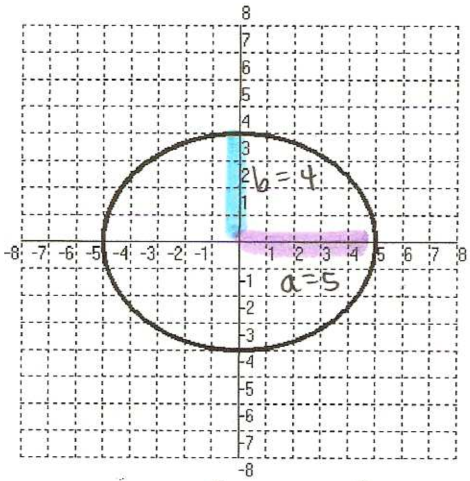
Instead of a single radius, an ellipse has two axes. The longer axis of an ellipse is the major axis and passes through both foci. The endpoints of the major axis are the vertices of the ellipse. The shorter axis of an ellipse if the minor axis. The endpoints of the minor axis are the co-vertices of the ellipse. The major axis and minor axis are perpendicular and intersect at the center of the ellipse.

Standard Form for the Equation of an Ellipse with Center at $(0,0)$		
Major Axis	Horizontal	Vertical
Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$
Vertices	$(a,0)$ and $(-a,0)$	$(0,a)$ and $(0,-a)$
Foci	$(c,0)$ and $(-c,0)$	$(0,c)$ and $(0,-c)$
Co-vertices	$(0,b)$ and $(0,-b)$	$(b,0)$ and $(-b,0)$
Graph		

The standard form of an ellipse centered at $(0,0)$ depends on whether the major axis is horizontal or vertical. The values of a, b, and c are related by the equation $c^2 = a^2 - b^2$. Also note that the length of the major axis is $2a$, the length of the minor axis is $2b$, and $a > b$.

Example 1: Write an equation in standard form for each ellipse with center $(0,0)$. Then find the domain and range.

a.

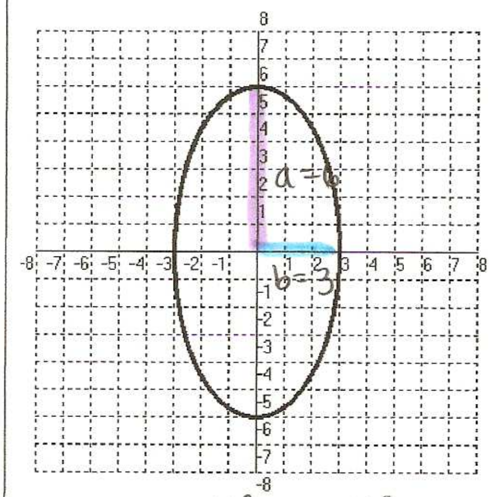


Equation: $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Domain: $-5 \le x \le 5$

Range: $-4 \le y \le 4$

b.



Equation: $\frac{x^2}{9} + \frac{y^2}{36} = 1$

Domain: $-3 \le x \le 3$

Range: $-6 \le y \le 6$

Example 2: Graph each ellipse. Find the foci as well, please. ☺

a. $\frac{x^2}{9}+\frac{y^2}{25}=1$

$b=3 \quad a=5$

vertices $(0, \pm 5)$

co-vertices $(\pm 3, 0)$

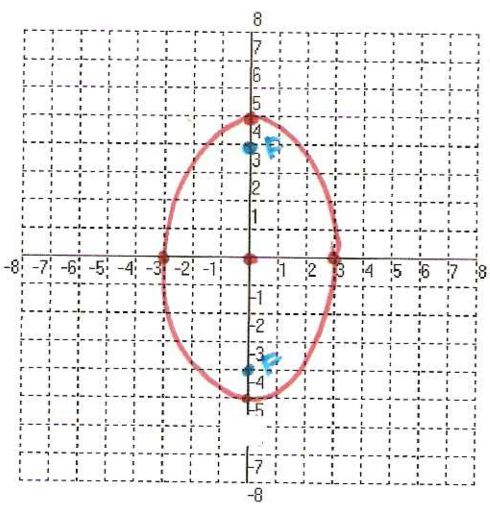
foci $(0, \pm 4)$

$c^2 = a^2 - b^2$

$c^2 = 25 - 9$

$c^2 = 16$

$c = \pm 4$



b. $4x^2 + 9y^2 = 36$

36

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$c^2 = 9 - 4$

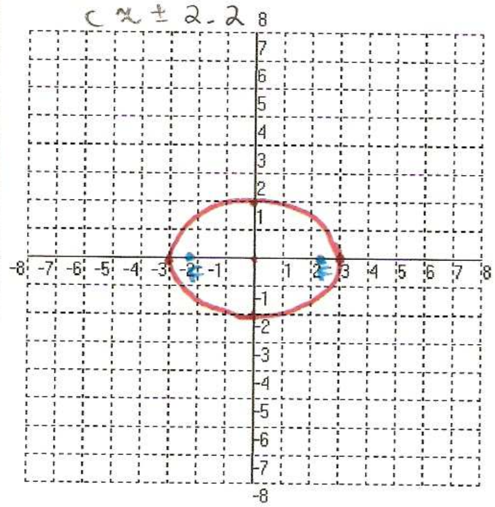
$c^2 = 5$

$c \approx \pm 2.2$

vertices $(\pm 3, 0)$

co-vertices $(0, \pm 2)$

foci $(\pm 2.2, 0)$



Ellipses may also be translated so that the center is NOT the origin...

Standard Form for the Equation of an Ellipse with Center at (h,k)		
Major Axis	Horizontal	Vertical
Equation	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
Vertices	$(h+a,k)$ and $(h-a,k)$	$(h,k+a)$ and $(h,k-a)$
Foci	$(h+c,k)$ and $(h-c,k)$	$(h,k+c)$ and $(h,k-c)$
Co-vertices	$(h,k+b)$ and $(h,k-b)$	$(h+b,k)$ and $(h-b,k)$

Example 3: Graph each ellipse.

a. $\frac{(x-3)^2}{16}+\frac{(y-1)^2}{36}=1$ center $(3,1)$

$b=4$ $a=6$

$c^2=a^2-b^2$

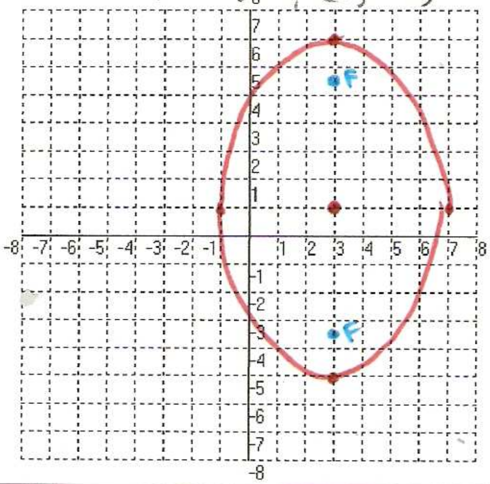
$c^2=20$

$c=\pm 4.5$

vertices $(3,7)$ & $(3,-5)$

co-vertices $(7,1)$ & $(-1,1)$

foci $(3,5.5)$ & $(3,-3.5)$



b. $\frac{(x+2)^2}{25}+\frac{(y-4)^2}{9}=1$ center $(-2,4)$

$a=5$ $b=3$

$c^2=a^2-b^2$

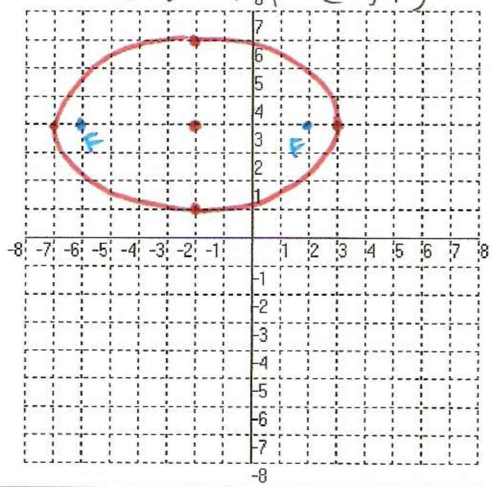
$c^2=16$

$c=\pm 4$

vertices $(3,4)$ & $(-7,4)$

co-vertices $(-2,7)$ & $(-2,1)$

foci $(2,4)$ & $(-6,4)$



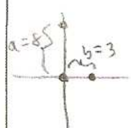
DAY TWO:

Example 4: Write an equation in standard form for each ellipse with center at $(0,0)$.

a. vertex $(0,8)$ and co-vertex $(3,0)$ center $(0,0)$

$a=8$

$b=3$



$$\boxed{\frac{x^2}{9}+\frac{y^2}{64}=1}$$

b. vertex $(-10,0)$ and focus $(8,0)$

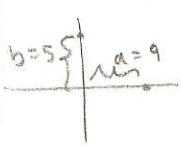
$c^2=a^2-b^2$

$64=100-b^2$

$b^2=36$

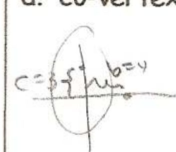
$$\boxed{\frac{x^2}{100}+\frac{y^2}{36}=1}$$

c. vertex $(9,0)$ and co-vertex $(0,5)$



$$\boxed{\frac{x^2}{81}+\frac{y^2}{25}=1}$$

d. co-vertex $(4,0)$ and focus $(0,3)$



$c^2=a^2-b^2$

$9=a^2-16$

$a^2=25$

$$\boxed{\frac{x^2}{16}+\frac{y^2}{25}=1}$$