Algebra 2 Notes
Section 10.2 - Circles

Name: _key_

A circle is the set of points in a ___plane___ that are a fixed ___distance___, called the ___radius___, from a fixed ___point___, called the ___center___. Because all of the points on a circle are the same ___distance___ from the center of the circle, you can use the ___Distance___ Formula to find the equation of a circle.
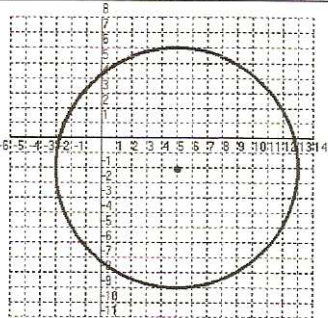
Example 1: Use the Distance Formula to write the equation of a circle with center $(2,1)$ and radius 5.

Remember that $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$$(5)^2 = \left(\sqrt{(x-2)^2 + (y-1)^2}\right)^2$$

$$\boxed{(x-2)^2 + (y-1)^2 = 25}$$

Not too much fun finding the equation of the circle with the Distance Formula, is it? Well, notice that ___r^2___ and the ___center___ are visible in the equation of a circle. This leads to a general formula for a circle with center ___(h, k)___ and radius ___r___.

Equation of a Circle

Equation	Example	Graph
The equation of a circle with center (h,k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.	The equation of the circle with center $(5,-2)$ and radius $r = 8$ is $(x-5)^2 + (y--2)^2 = 8^2$ OR $(x-5)^2 + (y+2)^2 = 64$	

Example 2: Write the equation of each circle.

a. circle with center $(0,-4)$ and radius 3

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-0)^2 + (y+4)^2 = 3^2$$
$$\boxed{x^2 + (y+4)^2 = 9}$$

b. circle with center $(-3,5)$ and containing the point $(9,-2)$
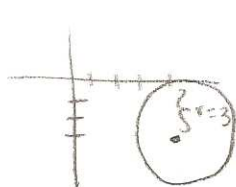
$$(x-h)^2 + (y-k)^2 = r^2$$
$$(9+3)^2 + (-2-5)^2 = r^2$$
$$12^2 + (-7)^2 = r^2$$
$$r^2 = 193$$
$$\boxed{(x+3)^2 + (y-5)^2 = 193}$$

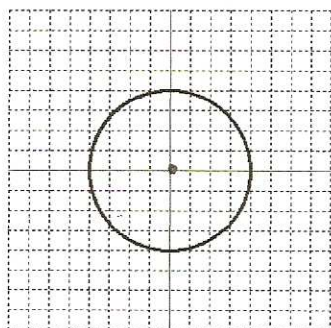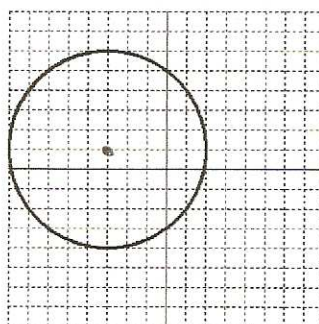| c. circle with the center $(4,-3)$ and tangent to the x-axis	d. circle with the center $(-2,5)$ and tangent to the y-axis
$(x-h)^2 + (y-k)^2 = r^2$ $(x-4)^2 + (y+3)^2 = 9$	$r=2$ $(x+2)^2 + (y-5)^2 = 4$
e. center $(0,0)$ $r=4$ $x^2 + y^2 = 16$	f. center $(-3,1)$ $r=5$ $(x+3)^2 + (y-1)^2 = 25$

Sometimes, you are given the equation of a circle... but the equation is NOT in the form $(x-h)^2 + (y-k)^2 = r^2$. Never fear! You can rewrite the equation in standard form by completing the square. ☺

Example 2:   Rewrite each circle in standard form. Then identify the center and the radius.

a. $x^2 + y^2 - 2x = 8$	b. $x^2 - 4x + y^2 + 6y = 12$
$x^2 - 2x + \underline{1} + y^2 = 8 + \underline{1}$ $(x-1)^2 + y^2 = 9$ center $(1,0)$ $r = 3$	$x^2 - 4x + \underline{4} + y^2 + 6y + \underline{9} = 12 + \underline{4} + \underline{9}$ $(x-2)^2 + (y+3)^2 = 25$ center $(2,-3)$ $r = 5$
c. $x^2 + y^2 - 10y - 24 = 0$	d. $x^2 + y^2 + 4x + 2y - 11 = 0$
$x^2 + y^2 - 10y + \underline{25} = 24 + \underline{25}$ $x^2 + (y-5)^2 = 49$ center $(0,5)$ $r = 7$	$x^2 + 4x + \underline{4} + y^2 + 2y + \underline{1} = 11 + \underline{4} + \underline{1}$ $(x+2)^2 + (y+1)^2 = 16$ center $(-2,-1)$ $r = 4$