Algebra 2 Notes Name: Key

Section 1.5 – Properties of Exponents

In an expression of the form a^n, a is the base, n is the exponent, and the quantity a^n is called a power. The exponent indicates the number of times that the base is used as a power.

Exponential Form	Base	Expanded Form	Result
$-2x^3$	x	$-2 \cdot x \cdot x \cdot x$	$-2x^3$
$-(2x)^3$	$2x$	$-(2x)\cdot(2x)\cdot(2x)$	$-8x^3$
$(-2x)^3$	$-2x$	$(-2x)(-2x)(-2x)$	$-8x^3$

Zero and Negative Exponents

For all nonzero real numbers a and b and integers n,

Words	Numbers	Algebra
Zero Exponent Property: A nonzero quantity raised to zero power is equal to 1.	$100^0 = 1$	$a^0 = 1$
Negative Exponent Property: A nonzero base raised to a negative exponent is equal to the reciprocal of the base raised to the opposite, positive exponent.	$7^{-2} = \left(\frac{1}{7}\right)^2 = \frac{1^2}{7^2} = \frac{1}{49}$ $\left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$	$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1^n}{a^n} = \frac{1}{a^n}$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

Example 1: Simplify each expression.

a. 2^{-3}

$\left(\frac{1}{2}\right)^3$

$\frac{1^3}{2^3} = \boxed{\frac{1}{8}}$

b. $-\left(\frac{3}{4}\right)^{-4}$

$-\left(\frac{4}{3}\right)^4$

$-\frac{4^4}{3^4} = \boxed{-\frac{256}{81}}$

c. $\left(\frac{1}{3}\right)^{-2}$

$\left(\frac{3}{1}\right)^2$

$\boxed{9}$

d. $(-5)^{-3}$

$\left(\frac{1}{-5}\right)^3$

$\frac{1^3}{(-5)^3} = \boxed{\frac{1}{-125}}$

Properties of Exponents:

For all nonzero real numbers a and b and integers m and n,

Words	Numbers	Algebra
Product of Powers Property: To multiply powers with the same base, add the exponents.	$4^3 \cdot 4^2 = 4^{3+2} = 4^5$	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers Property: To divide powers with the same base, subtract the exponents.	$\frac{3^7}{3^2} = 3^{7-2} = 3^5$	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power Property: To raise one power to another, multiply the exponents.	$\left(4^3\right)^2 = 4^{3\cdot 2} = 4^6$	$\left(a^m\right)^n = a^{m\cdot n}$
Power of a Product Property: To find the power of a product, apply the exponent to each factor.	$(3\cdot 4)^2 = 3^2 \cdot 4^2$	$(ab)^m = a^m b^m$
Power of a Quotient Property: To find the power of a quotient, apply the exponent to the numerator and denominator.	$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Now we will use the properties of exponents to simplify powers. **NOTE:** An algebraic expression is simplified when it contains no negative exponents, no grouping symbols, and no like terms.

Example 2: Simplify each expression. Assume all variables are nonzero.

a. $2x^3(-5x)$

$2(-5)\cdot x^3 \cdot x^1$

$-10 \cdot x^{3+1}$

$\boxed{-10x^4}$

b. $\left(\frac{2ab^4}{b^7}\right)^2$

$\frac{2^2 a^{1\cdot 2} b^{4\cdot 2}}{b^{7\cdot 2}}$

$\frac{4a^2 b^8}{b^{14}}$

$4a^2 b^{8-14}$

$4a^2 b^{-6}$

$\boxed{\frac{4a^2}{b^6}}$

c. $(5x^6)^3 \cdot x^{-20}$

$5^3 x^{6\cdot 3} \cdot x^{-20}$

$125 x^{18} \cdot x^{-20}$

$125 x^{18+-20}$

$125 x^{-2}$

$\boxed{\frac{125}{x^2}}$

d. $(-2a^3b)^{-3}$

$\left(\frac{1}{-2a^3b^1}\right)^3$

$\frac{1^3}{(-2)^3 a^{3\cdot 3} b^{1\cdot 3}}$

$\boxed{\frac{1}{-8a^9b^3}}$

e. $\left(\frac{5a^5b^2}{20a^{-2}b^2}\right)^{-3}$

$\left(\frac{20a^{-2}b^2}{5a^5b^2}\right)^3$

$\left(4a^{-2-5}b^{2-2}\right)^3$

$(4a^{-7}b^0)^3$

$4^3 a^{-7\cdot 3} b^{0\cdot 3}$

$64a^{-4}b^0$

$\boxed{\frac{64}{a^4}}$

f. $-5a^{-3}b^2(-12a^{-4}b^5c^{-2})^0$

$-5a^{-3}b^2 \cdot 1$

$\boxed{\frac{-5b^2}{a^3}}$

Scientific notation is a method of writing numbers by powers of 10. In scientific notation, a numbers takes the form $m\times 10^n$, where $1 \le m < 10$ and n is an integer.

Scientific Notation	Move the Decimal	Standard Notation
1.275×10^7	1.2750000.	12,750,000
3.5×10^{-1}	0.35	0.35

Example 3: Simplify each expression. Write the answer in scientific notation.

a. $\frac{9.1\times 10^{-3}}{1.3\times 10^8}$

$\frac{9.1}{1.3} \times 10^{-3-8}$

$\boxed{7 \times 10^{-11}}$

b. $(3.5\times 10^8)(5.2\times 10^5)$

$(3.5)(5.2) \times 10^{8+5}$

$18.2 \times 10^{13+1}$

$\boxed{1.82 \times 10^{14}}$