Algebra 2 Notes   Name: Key

Section 1.3 – Solving Linear Equations and Inequalities

An equation is a mathematical statement that two expressions are equivalent. The solution set of an equation is the value or values of the variable that make the equation true. Solving a linear equation requires isolating the variable on one side of the equation by using the properties of equality.

Properties of Equality		
Words	**Numbers**	**Algebra**
Addition If you add the same quantity to both sides of an equation, the equation will still be true.	$3=3$ $3+2=3+2$	$a=b$ $a+c=b+c$
Subtraction If you subtract the same quantity from both sides of an equation, the equation will still be true.	$3=3$ $3-2=3-2$	$a=b$ $a-c=b-c$
Multiplication If you multiply both sides of an equation by the same quantity, the equation will still be true.	$3=3$ $3(2)=3(2)$	$a=b$ $ac=bc$
Division If you divide both sides of an equation by the same quantity, the equation will still be true.	$3=3$ $\frac{3}{2}=\frac{3}{2}$	$a=b$ $\frac{a}{c}=\frac{b}{c}$ $\forall c\neq 0$

To isolate the variable, perform the inverse, or opposite, of every operation in the equation on both sides of the equation. Do inverse operations in the reverse order of the order of operations.

Example 1: **Solving Equations with Variables on ONE Side** Solve each equation.

a. $5(y-7)=25$

$\frac{5(y-7)}{5}=\frac{25}{5}$

$y-7=5$

$\quad +7 \quad +7$

$\boxed{y=12}$

b. $3y+2(y-4)=8$

$3y+2y-8=8$

$5y-8=8$

$\quad +8 \quad +8$

$\frac{5y}{5}=\frac{16}{5}$

$\boxed{y=\frac{16}{5}}$

c. $3(a+2)-4a=9$

$3a+6-4a=9$

$-a+6=9$

$\quad -6 \quad -6$

$\frac{-a}{-1}=\frac{3}{-1}$

$\boxed{a=-3}$

Example 2: **Solving Equations with Variables on BOTH Sides** Solve each equation.

a. $6y+21+7=4y-20+5y$

$6y+28=9y-20$

$-9y \quad -9y$

$-3y+28=-20$

$\quad -28 \quad -28$

$\frac{-3y}{-3}=\frac{-48}{-3}$

$\boxed{y=16}$

b. $3k-14k+25=2-6k-12$

$-11k+25=-6k-10$

$+6k \quad +6k$

$-5k+25=-10$

$\quad -25 \quad -25$

$\frac{-5k}{-5}=\frac{-35}{-5}$

$\boxed{k=7}$

c. $3(w+7)-5w=w+12$

$3w+21-5w=w+12$

$-2w+21=w+12$

$-w \quad -w$

$-3w+21=12$

$\quad -21 \quad -21$

$\frac{-3w}{-3}=\frac{-9}{-3}$

$\boxed{w=3}$

You have solved equations with a single solution. Equations may also have infinitely many solutions or no solution.

An equation that is TRUE for all values of the variable, such as $x = x$, is an identity. An equation that has no solution, such as $3 = 5$, is a contradiction because there are no values that make it true.

Example 3: **Identifying Identities and Contradictions** Solve each equation.

a. $3x+4x+5=7x+5$	b. $8(y+7)=6y-8+2y$	c. $3(2-3x)=-7x-2(x-3)$
$7x+5=7x+5$ $-7x \quad -7x$ $5=5$ true infinitely many sol'ns $\boxed{\mathbb{R}}$	$8y+56=8y-8$ $-8y \quad -8y$ $56=-8$ false no solution $\boxed{\varnothing}$	$6-9x=-7x-2x+6$ $6-9x=-9x+6$ $+9x \quad +9x$ $6=6$ true infinitely many sol'ns $\boxed{\mathbb{R}}$

An inequality is a statement that compares two expressions by using the symbols $<, >, \leq, \geq, \neq$. The graph of an inequality is the solution set, the set of all points on the number line that satisfy the inequality.

The properties of equality are true for inequalities, with one important difference. **IF YOU MULTIPLY OR DIVIDE BOTH SIDES OF AN INEQUALITY BY A** negative **NUMBER, YOU MUST** reverse (flip) **THE INEQUALITY SYMBOL!**

Example 4: **Solving Inequalities** Solve and graph each inequality.

a. $9x+4<12x-11$

$-12x \quad -12x$

$-3x+4<-11$

$-4 \quad -4$

$\frac{-3x}{-3} > \frac{-15}{-3}$ FLIP!

$\boxed{x>5}$

b. $4x+17\leq x+8$

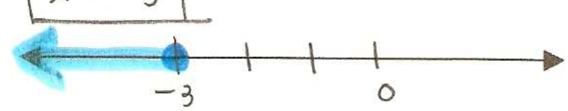
$-x \quad -x$

$3x+17\leq 8$

$-17 \quad -17$

$\frac{3x}{3} \leq \frac{-9}{3}$

$\boxed{x\leq -3}$



Example 5: Application

The local phone company charges $12.95 a month for the first 200 minutes of air time, plus $0.07 for each additional minute. If Nina's bill for the month was $14.56, how many additional minutes did she use?

m = # additional minutes

C = total charge

$C = 12.95 + 0.07m$

$14.56 = 12.95 + 0.07m$

$-12.95 \quad -12.95$

$\frac{1.61}{0.07} = \frac{0.07m}{0.07}$

$m = 23$

$\boxed{\text{23 add'l minutes}}$