

# Algebra 2 Notes

Name: Jenny

## Section 9.4 - Operations with Functions

You can perform operations on functions in much the same way that you perform operations on numbers or expressions. You can add, subtract, multiply, or divide functions by operating on their rules.

Notation for Function Operations	
Operation	Notation
Addition	$(f+g)(x) = f(x) + g(x)$
Subtraction	$(f-g)(x) = f(x) - g(x)$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ , where $g(x) \neq 0$

Example 1: Given  $f(x) = 2x^2 + 4x - 6$  and  $g(x) = 2x - 2$ , find each value.

a. $(f+g)(2)$ $f(2) + g(2)$ $f(2) = 2 \cdot 2^2 + 4 \cdot 2 - 6 = 10$ $g(2) = 2 \cdot 2 - 2 = 2$ $10 + 2 = \boxed{12}$	b. $(f-g)(-4)$ $f(-4) - g(-4)$ $f(-4) = 2(-4)^2 + 4(-4) - 6 = 10$ $g(-4) = 2(-4) - 2 = -10$ $10 - -10 = \boxed{20}$	c. $(fg)(-1)$ $f(-1) \cdot g(-1)$ $f(-1) = 2(-1)^2 + 4(-1) - 6 = -8$ $g(-1) = 2(-1) - 2 = -4$ $(-8)(-4) = \boxed{32}$
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Example 2: Given  $f(x) = 2x^2 + 4x - 6$  and  $g(x) = 2x - 2$ , find each function.

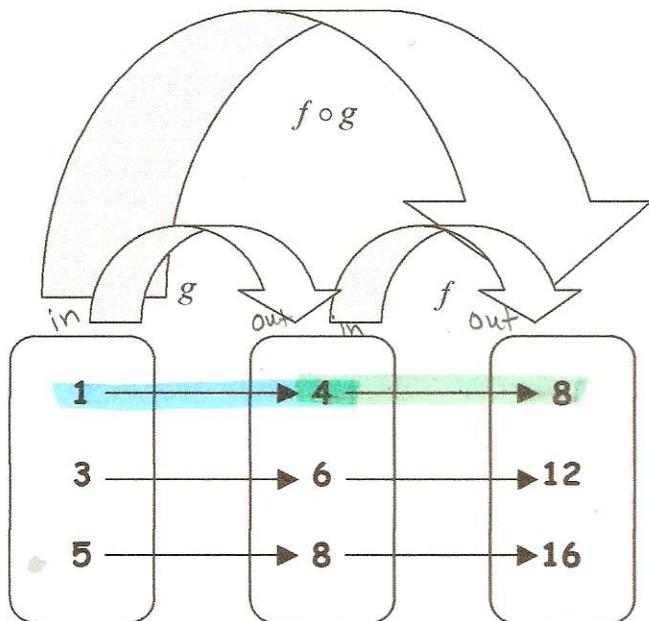
a. $(f+g)(x)$ $f(x) + g(x)$ $= 2x^2 + 4x - 6 + 2x - 2$ $= \boxed{2x^2 + 6x - 8}$	b. $(f-g)(x)$ $f(x) - g(x)$ $= 2x^2 + 4x - 6 - (2x - 2)$ $= 2x^2 + 4x - 6 - 2x + 2$ $= \boxed{2x^2 + 2x - 4}$
c. $(fg)(x)$ $f(x) \cdot g(x)$ $= (2x^2 + 4x - 6)(2x - 2)$ $= 4x^3 + 8x^2 - 12x - 4x^2 - 8x + 12$ $= \boxed{4x^3 + 4x^2 - 20x + 12}$	d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ $= \frac{2x^2 + 4x - 6}{2x - 2}$ $= \frac{2(x^2 + 2x - 3)}{2(x-1)}$ $= \frac{2(x+3)(x-1)}{2(x-1)} = \boxed{x+3}$ <small>where <math>x \neq 1</math></small>

Another function operation uses the output from one function as the input for a second function. This operation is called the composition of functions.

### Composition of Functions

The composition of functions  $f$  and  $g$  is notated  $(f \circ g)(x) = f(g(x))$ .

The domain of  $(f \circ g)(x)$  is all values  $x$  in the domain of  $g$  such that  $g(x)$  is in the domain of  $f$ .



To find  $(f \circ g)(1)$ , first find  $g(1)$ .

$$g(1) = 4$$

Then use  $4$  as the input into  $f$ :

$$f(4) = 8$$

$$\text{So } (f \circ g)(1) = f(g(1)) = 8$$

The order of function operations is the same as the order of operations for numbers and expressions. To find  $f(g(3))$ , evaluate  $g(3)$  first and then substitute the result into  $f$ .

Example 3: Given  $f(x) = 3x + 1$  and  $g(x) = x^3$ , find each value.

a.  $f(g(2))$

$$g(2) = 2^3 = 8$$

$$f(8) = 3(8) + 1$$

$$= 25$$

b.  $(g \circ f)(2)$

$$g(f(2))$$

$$f(2) = 3 \cdot 2 + 1 = 7$$

$$g(7) = 7^3$$

$$= 343$$

c.  $f(g(-3))$

$$g(-3) = (-3)^3 = -27$$

$$f(-27) = 3(-27) + 1$$

$$= -80$$

You can use algebraic expressions as well as numbers as inputs into functions. To find a rule for  $f(g(x))$ , substitute the rule for  $g$  into  $f$ .

Example 4: Given  $f(x) = 5x + 2$  and  $g(x) = \frac{2}{x-1}$ , write each composite function. State the domain of each.

a.  $f(g(x))$

$$f\left(\frac{2}{x-1}\right) = \frac{5}{1} \left(\frac{2}{x-1}\right) + 2 \\ = \boxed{\frac{10}{x-1} + 2}, \quad x \neq 1$$

domain:  $x \neq 1$

b.  $(g \circ f)(x)$

$$g(f(x)) = \\ g(5x+2) = \frac{2}{5x+2-1} \\ = \boxed{\frac{2}{5x+1}}, \quad x \neq -\frac{1}{5}$$

domain:  $x \neq -\frac{1}{5}$

Example 5: Use the tables to find each value.

$x$	1	2	3	4	5
$f(x)$	1	0	1	4	9

$x$	3	4	5	6	7
$g(x)$	0	2	4	6	8

a.  $(f+g)(4)$

$$f(4) + g(4)$$

$$4 + 2$$

b

b.  $\left(\frac{g}{f}\right)(5)$

$$\frac{g(5)}{f(5)} = \boxed{\frac{4}{9}}$$

c.  $(g \circ f)(4)$

$$g(f(4))$$

$$g(4)$$

2

d.  $(g-f)(5)$

$$g(5) - f(5)$$

$$4 - 9$$

-5

e.  $(fg)(3)$

$$f(3) \cdot g(3)$$

$$1 \cdot 0$$

0

f.  $(f \circ g)(4)$

$$f(g(4))$$

$$f(2)$$

0