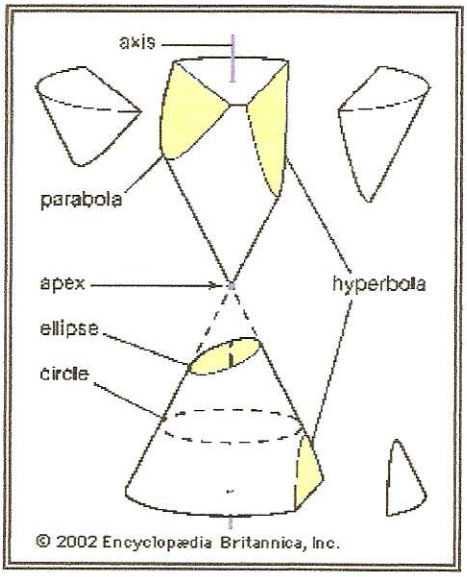
Algebra 2 Notes Name: Key

Section 10.6 – Identifying Conic Sections

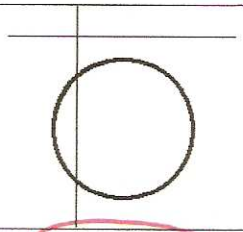
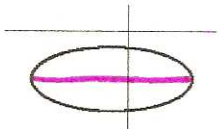
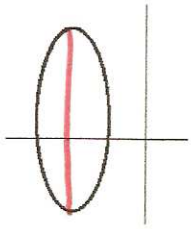
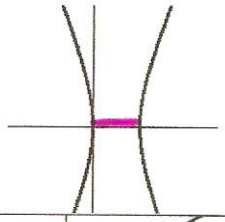
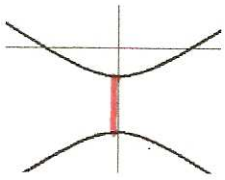
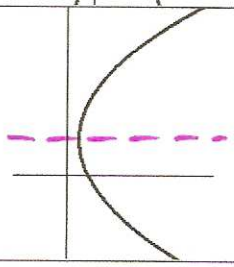
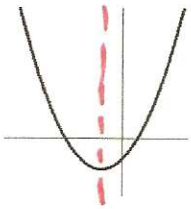


The conic sections result from intersecting a plane with a double cone, as shown in the figure. There are three distinct families of conic sections:

- the ellipse (including the circle)
- the parabola (with one branch)
- the hyperbola (with two branches)

Before we work in detail with these various conic sections, we are going to practice identifying them. ☺

Standard Form of Conic Sections:

Conic	Equation in Standard Form			
CIRCLE	$(x-h)^2+(y-k)^2=r^2$			
	Horizontal Axis		Vertical Axis	
ELLIPSE	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$		$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$	
HYBERBOLA	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$		$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$	
PARABOLA	$(y-k)^2=4p(x-h)$		$(x-h)^2=4p(y-k)$	

Example 1: Identify the conic section that each equation represents.

a. $\frac{x^2}{7^2}+\frac{y^2}{8^2}=1$ ellipse	b. $y^2=8(x+2)$ parabola	c. $\frac{(x-2)^2}{5^2}-\frac{(y-3)^2}{2^2}=1$ hyperbola
d. $(x+1)^2+(y-2)^2=3^2$ circle	e. $\frac{(y+3)^2}{4}-\frac{(x-1)^2}{9}=1$ hyperbola	f. $(x+5)^2=-4(y-2)$ parabola

You will not always be given the equation of a conic section in standard form, so you need to be able to identify the conic section based on some generalizations regarding their equations. The following table will help you determine the type of conic section if you have the equation set equal to zero.

Conic	Example	Generalization
CIRCLE	$4x^2+4y^2-3x+2y-12=0$	Sum of x^2 and y^2 w/ SAME Coefficients
ELLIPSE	$5x^2+4y^2-2x=0$ **OR** $3x^2+10y^2-x+y-10=0$	sum of x^2 and y^2 w/ DIFFERENT coefficients
HYBERBOLA	$x^2-y^2+2x-4y-2=0$ **OR** $4y^2-7x^2-3x-2y-15=0$	difference of x^2 and y^2 (or y^2 and x^2)
PARABOLA	$x^2-3x+4y+1=0$ **OR** $y^2+5x-6=0$	only x^2 or y^2, but not both

Example 2: Identify each conic section. Justify your answer.

a. $4x^2-9y^2-18y+27=0$ ✓ hyperbola — difference of x^2 and y^2	b. $1x^2+1y^2-4x+6y-36=0$ ✓ Circle — Sum of x^2 and y^2 w/ same coefficients
c. $x^2+3y^2+4x+6y+4=0$ ✓ ellipse — Sum of x^2 and y^2 w/ different Coefficients	d. $y^2+2y-3x-8=0$ parabola — only y^2, no x^2
e. $(2x)^2+4y^2=36$ $4x^2+4y^2-36=0$ ✓ Circle — sum of x^2 and y^2 w/ same coefficients	f. $3x^2+6x+10=3y^2-2y$ $3x^2-3y^2+6x+2y+10=0$ ✓ hyperbola — diff of x^2 and y^2
g. $x^2+2y^2-4x-6y+4=2y^2$ $x^2-4x-6y+4=0$ ✓ parabola — only x^2, no y^2	h. $5x^2+5y^2-2x^2-25=0$ $3x^2+5y^2-25=0$ ✓ ellipse — sum of x^2 and y^2 w/ diff. Coefficients