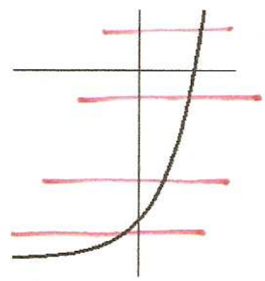
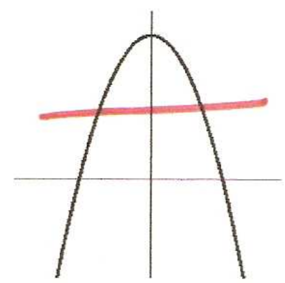
Algebra 2 Notes Name: Key

Section 9.5 – Functions and Their Inverses

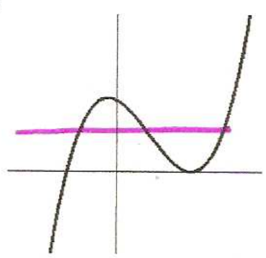
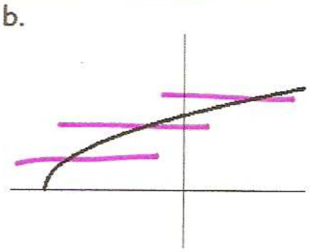
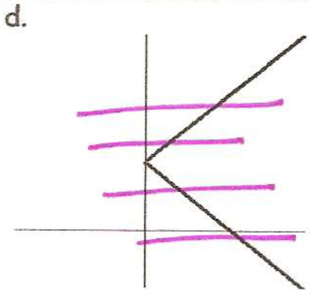
DAY ONE:

In Section 7.2, you learned that the inverse of a function $f(x)$ "undoes" $f(x)$. Its graph is a reflection across the line $y = x$. The inverse may or may not be a function.

Recall that the vertical - line test can help you determine whether a relation is a function. Similarly, the horizontal - line test can help you determine whether the inverse of a function is a function.

The Horizontal Line Test		
Words	**Examples**	
If any horizontal line passes through more than one point on the graph of a relation, the inverse relation is not a function.	 The inverse IS a function.	 The inverse IS NOT a function.

Example 1: Use the horizontal-line test to determine whether the inverse of each relation is a function.

a.	b.	c.	d.
 inverse is not a function	 inverse is a function	 inverse is a function	 inverse is a function

Recall from Section 7.2 that to write the rule for the inverse of a function, you can exchange x and y. Because the values of x and y are switched, the domain of the function will be the range of its inverse and vice versa.

Example 2: Find the inverse of each function. Determine whether it is a function, and state its domain and its range.

a. $f(x)=(x-2)^2-3$  fails HLT

$y=(x-2)^2-3$

inverse:

$x=(y-2)^2-3$

$\sqrt{x+3}=\sqrt{(y-2)^2}$

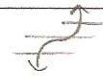
$y-2=\pm\sqrt{x+3}$

$\boxed{y=2\pm\sqrt{x+3}}$

not a function

D: $x\geq -3$

R: $\mathbb{R}$

b. $f(x)=x^3-3$  passes HLT

$y=x^3-3$

inverse:

$x=y^3-3$

$\sqrt[3]{x+3}=\sqrt[3]{y^3}$

$y=\sqrt[3]{x+3}$

$\boxed{f^{-1}(x)=\sqrt[3]{x+3}}$

function

D: $\mathbb{R}$

R: $\mathbb{R}$

DAY TWO:

You have seen that the inverses of functions are not necessarily functions. When both a relation and its inverse are functions, the relation is called a <u>One-to-one function</u>. In a one-to-one function, each y-value is paired with exactly one x-value.

You can use composition of functions to verify that two functions are inverses. Because inverse functions "undo" each other, when you compare two inverses the result is the input ________.

Identifying Inverse Functions		
Words	**Algebra**	**Example**
If the compositions of two functions equal the input value, the functions are inverses.	If $f(g(x))=g(f(x)=x)$, then $f(x)$ and $g(x)$ are inverse functions.	$f(x)=3x$ and $g(x)=\frac{1}{3}x$ $f(g(x))=3\left(\frac{1}{3}x\right)=x$ $g(f(x))=\frac{1}{3}(3x)=x$

Example 3: Determine by composition whether each pair of functions are inverses or not.

a. $f(x)=2x+4$ and $g(x)=\frac{1}{2}x-4$

$f(g(x))=f(\frac{1}{2}x-4)=2(\frac{1}{2}x-4)+4$

$=x-8+4$

$=x-4$

$\boxed{\text{not inverses}}$

b. For $x\geq 0$, $f(x)=\frac{1}{4}x^2$ and $g(x)=2\sqrt{x}$.

$f(g(x))=f(2\sqrt{x})=\frac{1}{4}(2\sqrt{x})^2$

$=\frac{1}{4}\cdot 4x$

$=x$

$g(f(x))=g(\frac{1}{4}x^2)=2\sqrt{\frac{1}{4}x^2}$

$=2\cdot\frac{1}{2}x$

$=x$ ✓

$\boxed{\text{inverses}}$