

Algebra 2 Notes

Section 7.1 - Curve Fitting with Polynomial Models

Name: Jay

The table shows the closing value of a stock index on the first day of trading for various years.

Year	1994	1995	1996	1997	2000	2001	2003	2004
Price (\$)	774	751	1053	1293	4186	2474	1347	2011

To create a mathematical model for the data, you will need to determine what type of function is most appropriate. In Section 5.8, you learned that a set of data that has constant Second differences can be modeled by a quadratic function. Finite differences can be used to identify the degree of any polynomial data.

Finite Differences of Polynomials		
Function Type	Degree	Constant Finite Differences
Linear	1	First
Quadratic	2	Second
Cubic	3	Third
Quartic	4	Fourth
Quintic	5	Fifth

Example 1: Use finite differences to determine the degree of the polynomial that best describes the data.

a.	b.	c.																																											
<table border="1"> <thead> <tr> <th>x</th><th>y</th></tr> </thead> <tbody> <tr> <td>-2</td><td>-10</td></tr> <tr> <td>-1</td><td>-4</td></tr> <tr> <td>0</td><td>-1.4</td></tr> <tr> <td>1</td><td>0</td></tr> <tr> <td>2</td><td>2.4</td></tr> <tr> <td>3</td><td>8</td></tr> </tbody> </table> <p>1st 2nd 3rd</p> <table border="1"> <thead> <tr> <th>x</th><th>y</th></tr> </thead> <tbody> <tr> <td>-6</td><td>-30</td></tr> <tr> <td>-4</td><td>15</td></tr> <tr> <td>-2</td><td>30</td></tr> <tr> <td>0</td><td>34</td></tr> <tr> <td>2</td><td>41</td></tr> <tr> <td>4</td><td>60</td></tr> </tbody> </table> <p>1st 2nd 3rd 4th</p> <table border="1"> <thead> <tr> <th>x</th><th>y</th></tr> </thead> <tbody> <tr> <td>-2</td><td>-24</td></tr> <tr> <td>-1</td><td>-27</td></tr> <tr> <td>0</td><td>-28</td></tr> <tr> <td>1</td><td>-21</td></tr> <tr> <td>2</td><td>0</td></tr> <tr> <td>3</td><td>41</td></tr> </tbody> </table> <p>1st 2nd 3rd</p>	x	y	-2	-10	-1	-4	0	-1.4	1	0	2	2.4	3	8	x	y	-6	-30	-4	15	-2	30	0	34	2	41	4	60	x	y	-2	-24	-1	-27	0	-28	1	-21	2	0	3	41	<p>The 3rd differences are constant.</p> <p><u>Cubic</u></p>	<p>The 4th differences are constant.</p> <p><u>quartic</u></p>	<p>The 3rd differences are constant.</p> <p><u>Cubic</u></p>
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Once you have determined the degree of the polynomial that best describes the data, you can use your calculator to create the function.

Example 2: Word Problem.

The table shows the population of a city from 1950 to 2000. Write a polynomial function for the data.

*	x	0	10	10	20	10	30	10	40	10	50
Year	1950	1960	1970	1980	1990	2000					
Population (thousands)	2853	4011	5065	6720	9704	14759					

Step 1: Find the finite differences of the y -values.

$$\begin{array}{cccccc}
 & 1158 & 1054 & 1655 & 2984 & 5055 \\
 1^{\text{st}} & & & & & \\
 & -104 & 601 & 1329 & 2071 & \\
 2^{\text{nd}} & & & & & \\
 & 705 & 728 & 742 & & \text{close} \\
 3^{\text{rd}} & & & & &
 \end{array}$$

Step 2: Determine the degree of the polynomial.

3rd differences are relatively close, so cubic polynomial should be a good model.

Step 3: Use the regression feature on your calculator to find the function model for the data.

* Let $x = \# \text{ years since } 1950$

$$y \approx 0.12x^3 - 4.21x^2 + 146.37x + 2851.64$$

Why not save ourselves some time? Often real-world data can be too irregular for you to use finite differences or find a polynomial function that fits perfectly. In these situations, you can use the regression feature of your graphing calculator. Make sure your Diagnostics on turned on. Then remember that the closer the R^2 -value is to 1, the better the function fits the data.

Example 3: Word Problem. Back to our original data table...

The table shows the value of a stock index on the first day of trading in various years. Use a polynomial model to estimate the value on the first day of trading in 2002.

*	x	0	1	2	3	6	7	9	10
Year	1994	1995	1996	1997	2000	2001	2003	2004	
Price (\$)	774	751	1053	1293	4186	2474	1347	2011	

* $x = \# \text{ years since } 1994$

linear $\rightarrow r^2 \approx .256$

quadratic $\rightarrow r^2 \approx .552$

cubic $\rightarrow r^2 \approx .628$

quartic $\rightarrow r^2 \approx .843$

closest to 1

$$y \approx 9.27x^4 - 191.56x^3 + 1168.22x^2 - 1702.58x + 999.60$$

* put EXACT equation into $y =$

$$2002 \rightarrow 2002 - 1994$$

$$x = 8$$

$$\text{plug in } 8 \rightarrow y \approx \$2030.95$$