

# Multiply Equivalent Matrices, Solving Matrix Equations

Show the steps to multiply each matrix without a calculator.

$$1. [3 \ -1] \cdot \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 8 \end{bmatrix}$$

$$2. \begin{bmatrix} 5 & 3 \\ -4 & 1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 26 \\ 18 & -14 \end{bmatrix}$$

$$3. \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$(3 \cdot 5) + (-1 \cdot 7) = 8 \\ 15 + -7$$

$$M_{11}: (5 \cdot -3) + (3 \cdot 4) = 3$$

$$M_{12}: (5 \cdot 4) + (3 \cdot 2) = 26$$

$$M_{21}: (-4 \cdot -3) + (1 \cdot 4) = 18$$

$$M_{22}: (-4 \cdot 4) + (1 \cdot 2) = -14$$

$$m_{11}: ap + br$$

$$m_{12}: aq + bs$$

$$m_{21}: cp + dr$$

$$m_{22}: cq + ds$$

Solve for each variable in the matrix.

4.

$$\begin{bmatrix} 16 & -8x \\ 3y & z \end{bmatrix} = \begin{bmatrix} 32/2 & 64 \\ 15 & 6z-20 \end{bmatrix}$$

$$\begin{aligned} -8x &= 64 & 3y &= 15 & z &= 6z-20 \\ x &= -8 & y &= 5 & z &= 4 \end{aligned}$$

5.

$$\begin{bmatrix} 3x-4 \\ x+y \end{bmatrix} = \begin{bmatrix} 10x+10 \\ 15 \end{bmatrix}$$

$$\begin{aligned} 3x-4 &= 10x+10 & x+y &= 15 \\ -14 &= 7x & -2+y &= 15 \\ -2 &= x & y &= 17 \end{aligned}$$

6.

$$\begin{bmatrix} 23 \\ 4y-3 \\ 16 \end{bmatrix} = \begin{bmatrix} 16-x \\ -3 \\ z^2 \end{bmatrix}$$

$$\begin{aligned} 23 &= 16-x & 4y-3 &= -3 & 16 &= z^2 \\ 7 &= -x & 4y &= 0 & z &= 4 \\ x &= -7 & y &= 0 & & \end{aligned}$$

Find the INVERSE of the matrix, if it is defined.

$$7. \begin{bmatrix} 1/2 & 0 \\ -1/6 & 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$$

$$8. \begin{bmatrix} 1 & 7 \\ 2 & 6 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{3}{4} & \frac{7}{8} \\ \frac{1}{4} & -\frac{1}{8} \end{bmatrix}$$

$$9. \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

Not possible

$$10. \begin{bmatrix} 8 & 7 \\ 9 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -7 \\ -9 & 8 \end{bmatrix}$$

Are the two matrices inverses? Why or why not?

$$11. \begin{bmatrix} 6 & -22 \\ -12 & 20 \end{bmatrix} \text{ and } \begin{bmatrix} -5/36 & -11/72 \\ -1/12 & -1/24 \end{bmatrix}$$

1) Must if  
[1 0]  
[0 1] yes

yes → 2) Inverse  
1 of them.

$$12. \begin{bmatrix} 6 & -7 & 3 \\ 0 & -4 & 1 \\ -1 & 11 & -2 \end{bmatrix} \text{ and } \begin{bmatrix} -6 & 7 & -3 \\ 0 & 4 & -1 \\ 1 & -11 & 2 \end{bmatrix}$$

NO

Solve the equations that contain matrices.

$$13. X + \begin{bmatrix} -3 \\ 9 \\ -2 \end{bmatrix} = \begin{bmatrix} 7 \\ -4 \\ -6 \end{bmatrix}$$

$$X = \begin{bmatrix} 10 \\ -13 \\ -4 \end{bmatrix}$$

$$14. 2X - \begin{bmatrix} -4 & 7 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 0 & -4 \end{bmatrix}$$

$$2X = \begin{bmatrix} 4 & 12 \\ 3 & -4 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 6 \\ \frac{3}{2} & -2 \end{bmatrix}$$

Solve each matrix equation using inverses.

15.  $\begin{bmatrix} -4 & 7 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 26 \\ -5 \end{bmatrix}$

$$A^{-1} \cdot A \cdot X = B$$

$$X = A^{-1} \cdot B$$

$$X = \begin{bmatrix} 95 \\ 58 \end{bmatrix}$$

16.  $\begin{bmatrix} 12 & 4 \\ -8 & 15 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 64 \\ -25 \end{bmatrix}$

$$A^{-1} \cdot A \cdot X = B$$

$$X = A^{-1} \cdot B$$

$$X = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

17.  $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 175 \end{bmatrix}$  N.P.

no solution

Rewrite the matrix equations (15-17) as a system of equations. (Hint: They will look like questions 21-26)

18.  $-4x + 7y = 26$   
 $3x + 2y = -5$

19.  $12x + 4y = 64$   
 $-8x + 15y = -25$

20.  $x - y = 5$   
 $-x + 7y = 175$

Write the matrix equation for the system. DO NOT SOLVE.

21.  $3x - y = 5$   
 $-2x + y = -4$

$$\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$$

22.  $4x - 3y = 15$   
 $-2x + 4y = -24$

$$\begin{bmatrix} 4 & -3 \\ -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ -24 \end{bmatrix}$$

23.  $5x + 9y = 1$   
 $-4x - 7y = 4$

$$\begin{bmatrix} 5 & 9 \\ -4 & -7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

24.  $2x + 4y = 3$   
 $y = -\frac{2}{3}x + \frac{1}{3}$

$$\begin{bmatrix} 2 & 4 \\ \frac{2}{3} & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ \frac{1}{3} \end{bmatrix}$$

25.  $13x - y = 8$   
 $-2x = -4$

$$\begin{bmatrix} 13 & -1 \\ -2 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

26.  $6x - 3y = 15$   
 $x + y = -2$

$$\begin{bmatrix} 6 & -3 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ -2 \end{bmatrix}$$