Algebra 2 Worksheet Name: Key
Section 6.5 - Finding Real Roots DAY TWO Period: ___

I. Use the Rational Root Theorem to identify all the POSSIBLE rational roots.

1. $f(x)=x^5-3x^4-5x+12$	2. $f(x)=3x^4+5x^2-7x+9$	3. $f(x)=4x^4-5x^3+x^2-x+1$
p: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$	p: $\pm1, \pm3, \pm9$	p: ±1
q: ±1	q: $\pm1, \pm3$	q: $\pm1, \pm2, \pm4$
$\boxed{\frac{p}{q}: \pm1, \pm2, \pm3, \pm4, \pm6, \pm12}$	$\boxed{\frac{p}{q}: \pm1, \pm3, \pm9, \pm\frac{1}{3}}$	$\boxed{\frac{p}{q}: \pm1, \pm\frac{1}{2}, \pm\frac{1}{4}}$

II. Use the Irrational Root Theorem to find the smallest possible degree of the polynomial with the given roots.

4. $6, \sqrt{3}, -\sqrt{5}$	5. $-4, 1-\sqrt{5}$	6. $1-\sqrt{2}, \sqrt{8}, 2+\sqrt{3}$
$-\sqrt{3}$ $+\sqrt{5}$	$1+\sqrt{5}$	$1+\sqrt{2}$ $-\sqrt{8}$ $2-\sqrt{3}$
$\boxed{5}$	$\boxed{3}$	$\boxed{6}$

III. Identify all the real roots of each equation. GIVE EXACT VALUES. No decimals.

7. $6x^3+67x^2-153x-130=0$ put into calc.

Find that -13 is a zero.

-13	6	67	-153	-130
		-78	143	130
	6	-11	-10	0

$6x^2-11x-10=0$ (-60)

mult $=-60$
add $=-11$
-15 & 4

$(3x+2)(2x-5)=0$

$3x+2=0$ $\quad$ $2x-5=0$

$3x=-2$ $\quad$ $2x=5$

$x=-\frac{2}{3}$ $\quad$ $x=\frac{5}{2}$

$\boxed{x=-\frac{2}{3}, \frac{5}{2}, -13}$

8. $x^4-9x^2+20=0$

$(x^2-5)(x^2-4)=0$

$x^2-5=0$ $\quad$ $x^2-4=0$

$\sqrt{x^2}=\sqrt{5}$ $\quad$ $\sqrt{x^2}=\sqrt{4}$

$x=\pm\sqrt{5}$ $\quad$ $x=\pm2$

$\boxed{x=\pm\sqrt{5}, \pm2}$

9. $16x^4 - 30x^3 - 175x^2 + 165x - 36 = 0$

From calc: -3 and 4 are roots

$$\begin{array}{r|rrrr|r} -3 & 16 & -30 & -175 & 165 & -36 \\ & \downarrow & -48 & 234 & -177 & 36 \\ \hline & 16 & -78 & 59 & -12 & 0 \end{array}$$

$$\begin{array}{r|rrr|r} 4 & 16 & -78 & 59 & -12 \\ & \downarrow & 64 & -56 & 12 \\ \hline & 16 & -14 & 3 & 0 \end{array}$$

$16x^2 - 14x + 3 = 0$ $\quad -6$ and -8 (48)

$(8x - 3)(2x - 1) = 0$

$8x - 3 = 0 \qquad 2x - 1 = 0$

$x = \frac{3}{8} \qquad x = \frac{1}{2}$

$\boxed{x = \frac{3}{8}, \frac{1}{2}, -3, 4}$

10. $10x^3 = 200x$

$10x^3 - 200x = 0$

$10x(x^2 - 20) = 0$

$10x = 0 \qquad x^2 - 20 = 0$

$x = 0 \qquad \sqrt{x^2} = \sqrt{20}$

$x = \pm 2\sqrt{5}$

$\boxed{x = 0, \pm 2\sqrt{5}}$

11. ~~$x^4 - x^3 - 18x^2 + 12x + 72$~~

$x^4 - x^3 = 18x^2 - 12x - 72$

$x^4 - x^3 - 18x^2 + 12x + 72 = 0$

From calc: -2 and 3 are roots

$$\begin{array}{r|rrrr|r} -2 & 1 & -1 & -18 & 12 & 72 \\ & \downarrow & -2 & 6 & 24 & -72 \\ \hline & 1 & -3 & -12 & 36 & 0 \end{array}$$

$$\begin{array}{r|rrr|r} 3 & 1 & -3 & -12 & 36 \\ & \downarrow & 3 & 0 & -36 \\ \hline & 1 & 0 & -12 & 0 \end{array}$$

$x^2 - 12 = 0$

$\sqrt{x^2} = \sqrt{12}$

$x = \pm 2\sqrt{3}$

$\boxed{x = -2, 3, \pm 2\sqrt{3}}$

12. $x^3 - 15x^2 = -43x - 11$

$x^3 - 15x^2 + 43x + 11 = 0$

From calc: 11 is a root

$$\begin{array}{r|rrrr|r} 11 & 1 & -15 & 43 & 11 \\ & \downarrow & 11 & -44 & -11 \\ \hline & 1 & -4 & -1 & 0 \end{array}$$

$1x^2 - 4x - 1 = 0$

$a = 1 \quad b = -4 \quad c = -1$

$x = \frac{4 \pm \sqrt{16 - 4(-1)}}{2}$

$x = \frac{4 \pm \sqrt{20}}{2}$

$x = \frac{4 \pm 2\sqrt{5}}{2}$

$x = 2 \pm \sqrt{5}$

$\boxed{x = 11, 2 \pm \sqrt{5}}$

IV. Graphing Calculator.

13. Consider the polynomial function $f(x) = x^4 + 3x^3 - 3x^2 - 12x - 4$.

(a) Use the Rational Root Theorem to list the possible rational roots of this equation.

$p: \pm 1, \pm 2, \pm 4$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

(b) Graph the polynomial on a graphing calculator. Which possible rational roots are zeros of $f(x)$? How do you know?

$2, -2$ $\quad$ x-intercepts of function

(c) According to the graph, how many other real zeros does the function have? 2

(d) Approximate these zeros to the nearest hundredth by using the zero feature. ≈ -2.62 and -0.38